Needham, Chapter 8, Exercise 6, p. 420.
G. Palmer, June 20, 2017, 3 pm PST

6 Confirm by parametric evaluation that the integral of z^m round an origin-centred circle vanishes, except when m = -1.

Let z = re^{it}. Then $z^m = r^m e^{imt}$, $dz = r\, i\, e^{it}$ dt.

$$\oint_C z^m \, dz = \int_0^{2\pi} r^m e^{imt} \, r\, i\, e^{it} \, dt$$

$$= r^{m+1}\, i \int_0^{2\pi} e^{i(m+1)t} \, dt$$

$$= r^{m+1} \frac{e^{i(m+1)t}}{m+1} \Big|_0^{2\pi}$$

$$= \frac{r^{m+1}}{m+1}\left(e^{i(m+1)2\pi} - e^0\right) = \frac{r^{m+1}}{m+1} (1-1) = 0$$

When m = -1, this is the same problem as Exercise 4 (i):

(i) $|z| = 1$

Parametric evaluation following method in § IX, p. 409:

Let $z(t) = e^{it}$. Then $v(t) = ie^{it}dt$.

$$\oint_C (1/z)\, dz = \int_0^{2\pi} \frac{v(t)}{z(t)} dt = \int_0^{2\pi} \frac{1}{e^{it}}\, ie^{it}\, dt = i \int_0^{2\pi} dt = it \Big|_0^{2\pi} = 2\pi i$$