

Needham, Chapter 8, Exercise 3, p. 420.

G. Palmer, June 12, 2017, 2:30 pm PST

3 In the integration of the real function $f(x)$, let L denote the length of the integration range, and let M denote the maximum size of $f''(x)$ in this range. From the previous two exercises, deduce the standard result,

$$\text{total Trapezoidal error} < \frac{1}{12} LM\Delta^2.$$

Note that this M is eight times the M on p. 382. We know from Ex. 1 that $\text{area}(ABCD) = \frac{1}{8} f''(x) \Delta^3$, and we know from Ex. 2 that limit of quotient of the area between the chord and the curve L over the area between the tangent and L is 2. The total trapezoidal error is just the area between the chord and the curve, which is $2/3 \text{ area}(ABCD)$. Then, substituting M for $f''(x)$ and multiplying by the number of errors (L/Δ)

$$\text{total TR error} = (2/3) (L/\Delta) (1/8) M \Delta^3$$

$$= \frac{1}{12} LM\Delta^2$$

Likewise, deduce the somewhat less familiar result,

$$\text{total } R_M \text{ error} < \frac{1}{24} LM \Delta^2$$

The total R_M error is just the area between the tangent and the curve, which is half the area between the chord and the curve. Therefore, the second equation is correct.