Needham, Chapter 8, Exercise 24, p. 425.
G. Palmer, August 23, 2017, 3:30 pm PST

24 Let K be a closed contour, and lt ν be its winding number about the point a. Show that

$$\oint_K \left(\frac{e^z}{z-a}\right) dz = 2\pi i \nu\, e^a.$$

[Hint: Write e^z as $e^a\, e^{(z-a)}$, and expand $e^{(z-a)}$ as a power series.] This is a special case of Cauchy's Integral Formula (explained in the next chapter), which states that if f is analytic inside K, then

$$\oint_K \left(\frac{f(z)}{z-a}\right) dz = 2\pi i\ \nu.$$

$$e^a\, e^{(z-a)} = e^a\left(1 + (z\text{-}a) + \frac{(z-a)^2}{2} + \frac{(z-a)^3}{6} + \cdots\right)$$

$$\frac{f\,(z)}{(z-a)} = \frac{e^a\left(1+(z-a)+\frac{(z-a)^2}{2}+\frac{(z-a)^3}{6}+\cdots\right)}{z-a}$$

$$= e^a\left(\frac{1}{z-a} + 1 + \frac{(z-a)}{2} + \frac{(z-a)^2}{6} + \cdots\right)$$

$$\oint_K \left(\frac{f(z)}{z-a}\right) dz = e^a \oint_K \left(\frac{1}{z-a} + 1 + \frac{(z-a)}{2} + \frac{(z-a)^2}{6} + \cdots\right) dz$$

$$= e^a \oint_K \left(\frac{1}{z-a}\right) dz = 2\pi\, i\ \nu\ e^a$$

by Cauchy's Residue Theorem (p. 401) and (7), p. 392