Needham, Chapter 7, Exercise 15, p. 374
G. Palmer, December 13, 2017, 10:20 am PT

15 The version of Brouwer's Fixed Point Theorem established in the text fell short of the full result in two ways: (A) we assumed that |g| < 1 on D rather than |g| ≤ 1; (B) we essentially used the Topological Argument Principle, which the previous exercise shows to be useless in the general case of a continuous mapping having infinitely many p-points in a finite region. Let's remove these blemishes. Once again let m(z) = g(z) - z be the movement of z, and suppose that Brouwer's result is false, so that m ≠ 0 throughout the disc |z| ≤ 1. Obtain the desired contradiction as follows:

Vasco (Forum, personal communication, 12/13/17) has provided a distillation of the problem as follows:

I would summarise the overall argument as follows:
Let g(z) be a continuous mapping of the unit disc D to itself such that |g(z)| ≤ 1.
Assume BFPT to be false, or in other words, that m(z) ≠ 0 throughout the unit disc D where z ≤ 1.
(i) We show that it follows from the above that ν[m(C), 0] = 1
(ii) We show that it follows from the same assumptions as in (i), but by using a different line of reasoning, that ν[m(C),0] = N, where N is any integer, positive or negative.
Since (i) and (ii) lead to mutually contradictory assertions, our ini-
tial assumption that BFPT is false must be wrong and so BFPT must be true.

He points out that the problem is an example of proof by contradiction, which is explained in Wikipedia <https://en.wikipedia.org/wiki/Proof_by_contradiction#Principle>.

I think it is also important to bear in mind, that ν[m(C), 0] here refers to ν[m(C), z] as z traverses C with radius less than or equal to 1. To write ν[m(C), 0], we tranpose the origin of the vector m to the origin of the complex plane.

Needham's problem statement, continued:

(i) By assumption, m(z) = g(z) - z = g(z) + f(z) does not vanish on the unit circle C. Use Ex. 12(iii) to show that if |g| ≤ 1 then ν[m(C), 0] = 1.

We assume that z traverses C_1. We want ν[H(z), 0] = 0, where H(z) = 1 + g(z)/f(z)], so that ν[f+g, 0] = ν[f, 0]. The equations are satisfied if g + f ≠ 0 and |g| ≤ |f|, as in 12(iii). It Is given that m(z) = (g + f) ≠ 0 throughout the disc |z| ≤ 1. Then for |g| ≤ (1 = |f(z)|),

$$\nu[f+g, 0] = \nu[f, 0] + \nu[H(z), 0]$$

$$= 1 + 0 = 1$$

(ii) Let C_r be the circle $|z| = r$, so that $C_1 = C$. By considering the evolution of $\nu[m(C_r), 0]$ as r increases from 0 to 1, obtain a contradiction with (i).

As r increases from 0 to 1 with $|g| <= 1$, m may make any number of winds around z, so $\nu[m(C_r),\ 0] = N$. In the limit where r = 1, m can not actually wind around z, so the winding number must be 0 or undefined. Hence

$$(\nu[m(C_r),\ 0] = \text{N}) \text{ or } \nu[m(C_r),\ 0] = (0 \text{ or undefined})$$

All of these possibilities contradict (i). Hence, the BFPT is NOT false as assumed, so it must be true.