

Needham, Chapter 7, Exercise 10, p. 372.
 G. Palmer, April 20, 2017, 2:30 pm PST

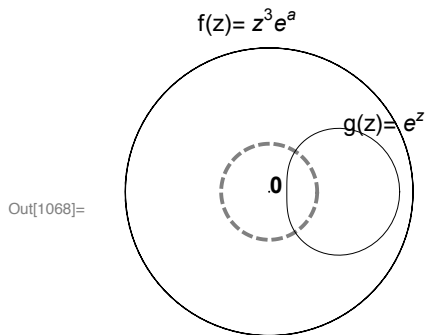


Figure 1:

Rouche's theorem: $z^n e^a = e^z$

10 If a is real and greater than 1, use Rouché's Theorem to show that the equation

$$z^n e^a = e^z$$

has n solutions inside the unit circle.

Let $f(z) = z^n e^a$ and $g(z) = e^z$. Find $|f(z)|$ and $|g(z)|$ on $|z| = 1$.

$$|f(z)| = |z^n e^a| = |e^a| > e$$

$$|g(z)| = |e^z| = |e^{x+iy}| = |e^x e^{iy}| = e^x \leq e$$

Then $|g(z)| < |f(z)|$. $(f+g)$ has the same number of roots as f , so there are n solutions inside the unit circle.