Needham, Chapter 6, Exercises 2, p. 328
G. Palmer, August 7, 2016, 10:55 pm PST

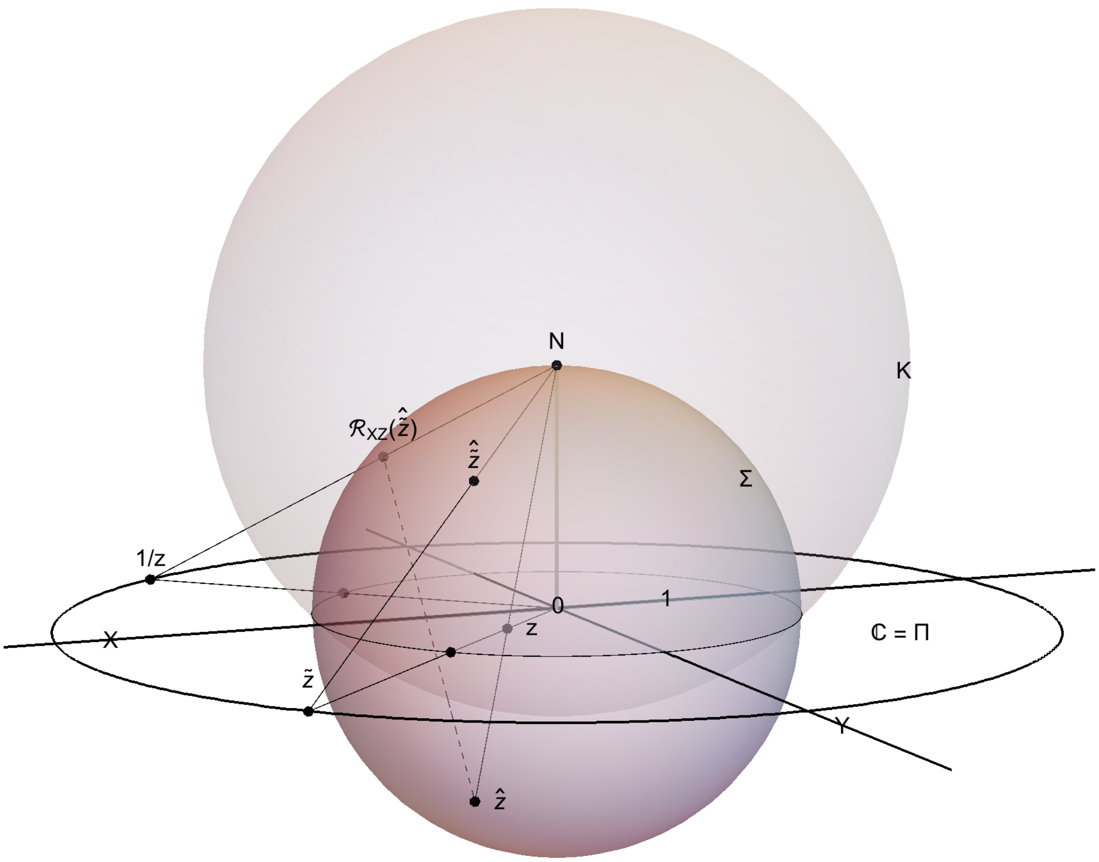

Figure 1. Inversion in unit circle in ℂ induces
stereographic projections restricted to Σ and ℂ

Preliminaries to Exercise 2. In Figure 1, we see that z induces $\hat{z}$ by stereographic projection. Inversion of z in the unit circle on ℂ sends z to $\tilde{z}$, which induces $\hat{\tilde{z}}$ by stereographic projection. The points $\hat{z}$ and $\hat{\tilde{z}}$ represent a reflection of Σ in its equatorial plane ℂ. Hence, $\mathcal{I}_C$ induces reflection of Σ in ℂ. We have illustrated (17), p. 143.

While on the topic and having a figure with the needed features, we also examine the derivation of (18). In Figure 1, we see that inversion in the unit circle followed by conjugation sends z to 1/z, as in Figure [21b], p. 142. Inversion in the unit circle is conceptualized as inversion in the vertical cross-section of Σ, but it is also inversion in the unit circle on ℂ.

Then conjugation sends $\tilde{z}$ to 1/z. Points on the unit circle are sent to the unit circle. Theorem (18), p. 143, claims that *the mapping z ⟶ 1/z in ℂ induces a rotation of the Riemann sphere about the real axis* *through an angle of π.* This is hard to interpret. It appears that the point $\mathcal{R}_{xy}(\hat{\tilde{z}})$, induced by 1/z, can be obtained by rotating the induced point $\hat{z}$, induced by z, about the real axis through an angle of π. One could also get there by “the composition of two reflections in perpendicular planes *through the real axis*” [emphasis mine]. That is, the planes (say XZ and XY) intersect at the real axis. The reflections are across the planes, not across the axis itself. Reflect $\hat{z}$ to $\hat{\tilde{z}}$ across XY, and then reflect $\hat{\tilde{z}}$ to $\mathcal{R}_{xz}(\hat{\tilde{z}})$.