Needham, Chapter 6, Exercise 1, p. 326.
G. Palmer, July 3, 2016

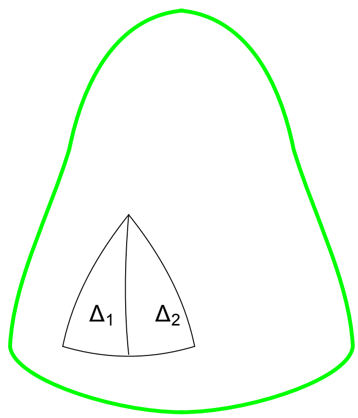

Figure 1. Hypothetical geodesic triangle on fruit

1 Draw a geodesic triangle Δ on the surface of a suitable fruit or vegetable [Figure 1]. *Now draw a geodesic segment from one of the vertices to an arbitrary point of the opposite side [of the triangle]. This divides Δ into two geodesic trangles, say Δ_1 and Δ_2. Show that the angular excess function E is additive, i.e. $E(\Delta) = E(\Delta_1) + E(\Delta_2)$. By continuing this process of subdivision, deduce that (5) implies (6).*

Vasco's proof goes something like this:

Let σ be the sum of angles of the large triangle. Then $E(\Delta) = \sigma$ - Pi, because Pi is the sum of angles of an Euclidean triangle. Let σ_1 and σ_2 be the sums of the angles of Δ_1 *and* Δ_2. The geodesic segment that splits the opposite side of Δ forms two angles that sum to Pi. Then $\sigma = \sigma_1 + \sigma_2$ - Pi and σ - Pi = (σ_1 - Pi) + (σ_2 - Pi). Hence, $E(\Delta) = E(\Delta_1) + E(\Delta_2)$.

This subdivision can be repeated on random vertices and their opposite sides ad infinitum. Then, for these infinitesimal triangles

$$E(\Delta) = E(\Delta_1) + E(\Delta_2)$$

$$= \mathrm{k(p)\ d}A_{\Delta_1} + \mathrm{k(p)\ d}A_{\Delta_2}$$

$$E(T) = \sum k(p)\, dA, \text{ over all the infinitesimal triangles}$$

$$= \iint_T k(p) dA$$