Needham, Chapter 5, Exercise 9, p. 258.
G. Palmer, January 4, 2015

9. Derive the quotient rule in an analogous way to the product rule (see page 226). [Hint: Multiply top and bottom of (A/B) by (b-ξg').]

$A = a+\xi f'$ and $B = b+\xi g'$

$A/B = (a+\xi f')/(b+\xi g') = (a+\xi f')/(b+\xi g')\ (b-\xi g')/(b-\xi g')$

$= [(a+\xi f')(b-\xi g')]/[(b+\xi g')(b-\xi g')]$

$= (ab - a\xi g' + b\xi f' - \xi^2 f' g')\big/(b^2 - \xi^2 g'^2)$

$\simeq (ab - \xi(ag' - bf'))/b^2$, by ignoring the infinitesimal products

$= (ab + \xi(bf' - ag')/b^2$ (where $\xi(bf' - ag')/b^2$ is the image of ξ)

From this we deduce the quotient rule:

$(f/g)' = (bf' - ag')/b^2$

By construction, f = a and g = b (Figure 1). Then

$(f/g)' = (gf' - fg')\big/g^2$

which is the quotient rule (Stewart J. *Calculus*, 2nd Ed., Brooks/Cole, Pacific Grove, p. 120).

Figure 1 plots some of the points and vectors involved using $f = \log|z| + i\mathrm{Arg}(z)$ and $g = z^2$. It also shows that f, g, and f/g are conformal.

Out[29]=

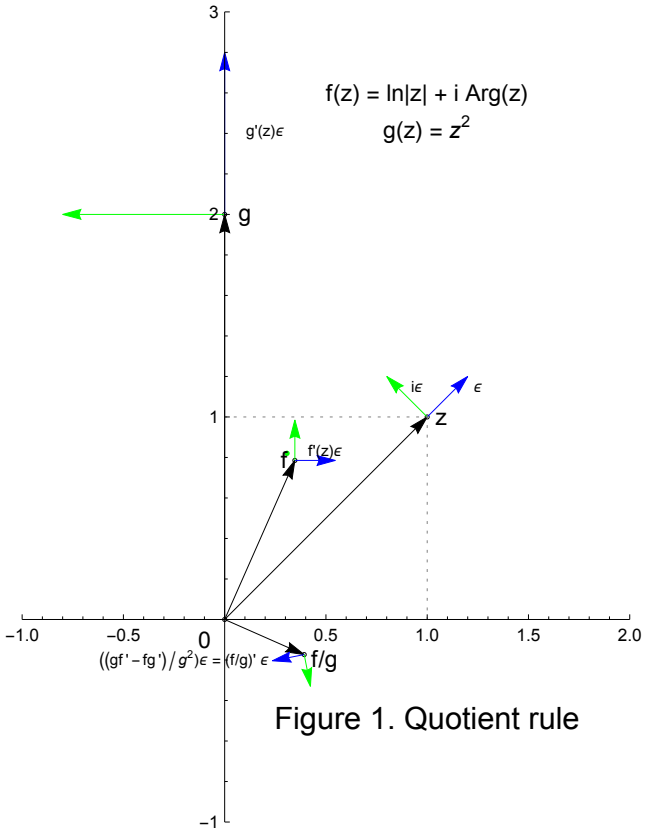


Figure 1. Quotient rule