Needham, Chapter 5, Exercise 8, p. 258.
G. Palmer, January 1, 2016

8. Calculate, then draw on a picture, a possible location for log(1+i). Draw a small shape at 1 + i. Use the amplitwist of log(z) to draw its image. Verify this using your computer.

Figure 1 was plotted prior to reading Vasco's explanation of his plot. I subsequently added an explanation and dotted lines at ln $\sqrt{2}$ and iPi/4. We can see by inspection that the angle of (1 + i) is Pi/4. We drew the blue vector $\tilde{\epsilon}$ at 1+i with the same angle. It forms the first side of the square at 1+i. The green vector at 1+i is $i\tilde{\epsilon}$. The four sides of the square were plotted by multiplying $\vec{\epsilon}$ by successive powers of i and adding the result to the endpoint of the previous side. The point at log(1+i) can be calculated with log(z) = ln|z| + iθ (p. 98).

$$\log(1+i) = \ln \sqrt{2} + iPi/4$$

The amplitwist of log(1+i) is 1/(1+i) (p. 222). If we represent (1+i) as $\sqrt{2}\ e^{iPi/4}$, then the amplitwist is $\frac{\sqrt{2}}{2}\ e^{-iPi/4}$. Since Arg($\vec{\epsilon}$) = Pi/4 and the twist is -Pi/4, we can write

$$\text{Arg}(f'(z)\ \vec{\epsilon}) = \text{Arg}(\frac{\sqrt{2}}{2}\ e^{-iPi/4}\ r_\epsilon e^{iPi/4}) = 0,$$

which is the angle of the blue arrow in the smaller square based at log(1+i). From the dotted lines and corresponding colors, we can see that the right angles have been preserved, so log(z) is conformal.

Out[4560]=

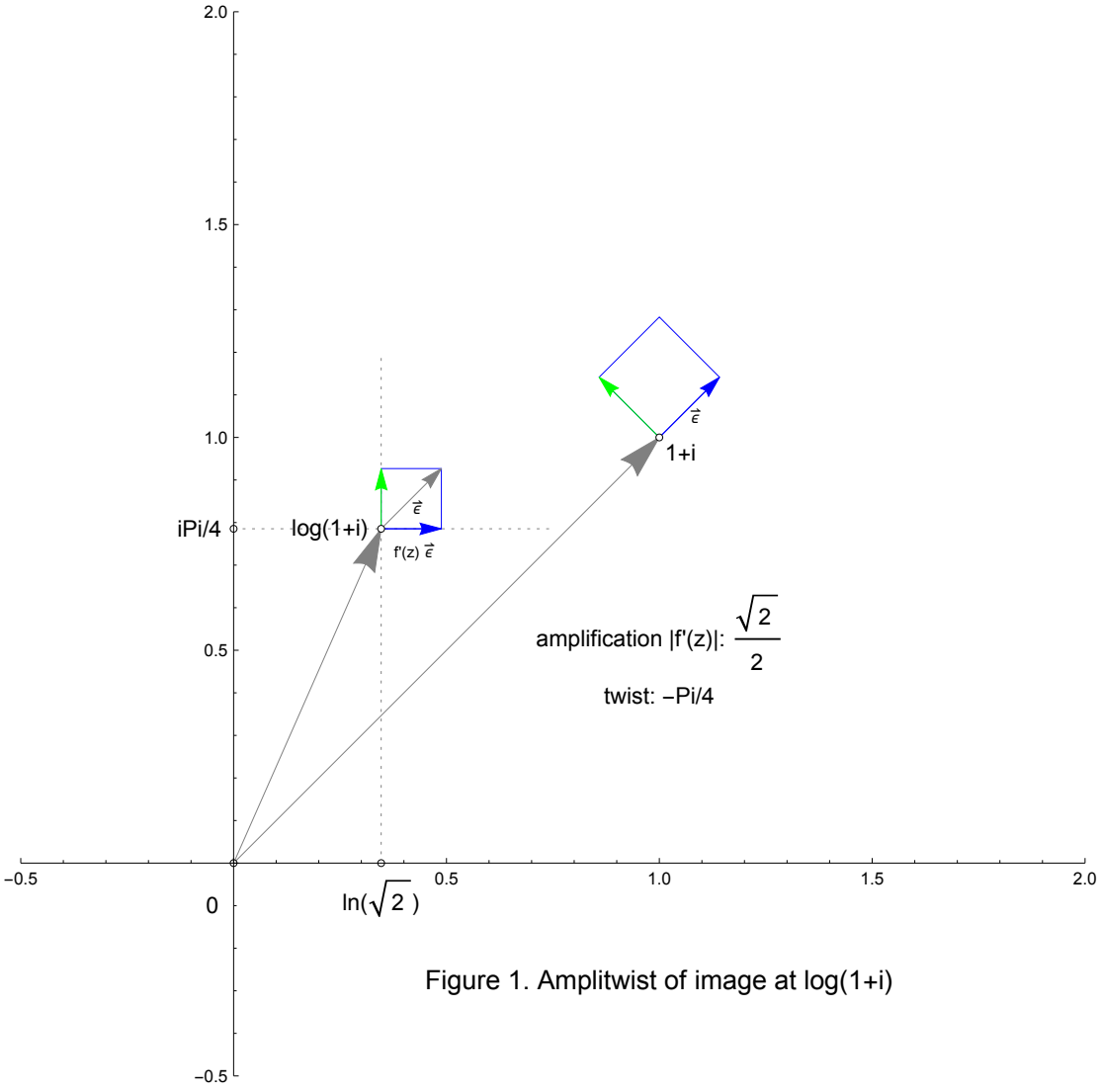


Figure 1. Amplitwist of image at log(1+i)