Needham, Chapter 5, Section IX, p. 235
G. Palmer, December 16, 2015

Application of Figure [17] to illustrate curvature and circle of curvature. Objects are created for two figures. The second figure is transposed to the right side of Figure 1.

It was more difficult than I anticipated to create this plot, and I am not sure that it is correct. For the curvature on the right hand side of Figure 1, I used the equation

$$\tilde{k}(\hat{\xi}) \equiv \text{Im}\left[\frac{f''\hat{\xi}}{f'|f'|}\right], \quad \text{where } \hat{\xi} = \frac{\vec{\xi}}{|\vec{\xi}|}, \text{ the unit tangent at p, and } \vec{\xi} \text{ and } \hat{\xi} \text{ are complex numbers.}$$

$\tilde{k}(\hat{\xi})$, a scalar, is just the curvature added by the mapping e^z. The original curvature of the slanted line is zero.

To find the center of the circle, multiplication of a small vector parallel to the tangent at p by f'(p) reveals the amplitwist at f(p):

$$f'(p)\,\vec{\xi} = e^{p}\vec{\xi}$$

Then, the length of the teal colored arrow represents the curvature. The red arrow is plotted in the direction of the center of the circle of curvature from f(p). This direction is arg(f'(p) $\vec{\xi}$) + Pi/2, and is implemented in the expression $i\tilde{k}(\vec{\xi})\tilde{\xi}/|\tilde{\xi}|$.* The radius of the circle of curvature is $1/\tilde{k}(\vec{\xi})$.

*See the "kArrow" definition in the (*CURVATURE*) section of the sourcecode. The $\tilde{\xi}$*'s* are complex vectors. *Mathematica* resists stacking of overset diacriticals, so I have omitted the vector arrow and have not attempted to use the hat for the unit vector.

Out[541]=

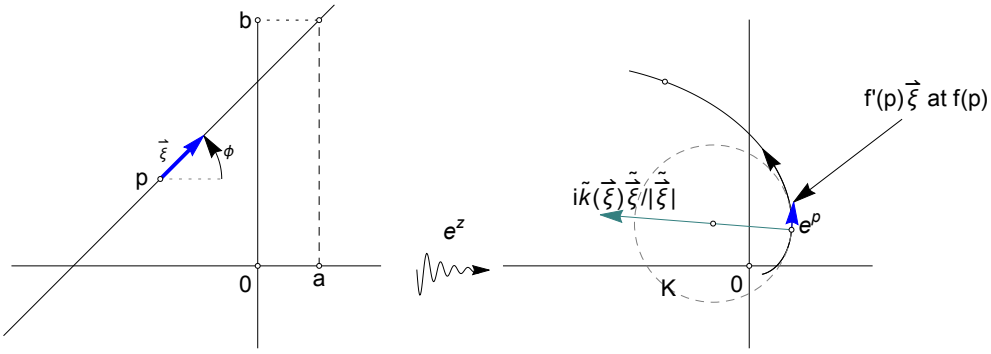


Figure 1. Circle of curvature K at p under e^z on straight line

$\tilde{k}(\vec{\xi})$=Im[(1/|e^p|)($\vec{\xi}$/|$\vec{\xi}$|)]: 1.56255

r=1/|$\tilde{k}(\vec{\xi})$|