**Derivation of Polar-Polar form of Cauchy-Riemann equations, Neeham, p. 219.**
**G. Palmer, December 2, 2015**

**"Verify that if we write $f = Re^{i\Psi}$... then CR takes the form.**

$$\partial_\theta(R) = -\,rR\partial_r(\Psi), \quad \text{and} \quad R\partial_\theta(\Psi) = r\,\partial_r(R)"$$

$$f_r' = \partial_r(R)\,e^{i\Psi} + iRe^{i\Psi}\partial_r(\Psi) \qquad (1)$$

$$f_\theta' = \partial_\theta(R)\,e^{i\Psi} + iRe^{i\Psi}\partial_\theta(\Psi) \qquad (2)$$

$$\partial_\theta f = ir\partial_r f \qquad (3),\ p.\ 218$$

Substitute into (3) from (1) and (2).

$$\partial_\theta(R)\,e^{i\Psi} + iRe^{i\Psi}\partial_\theta(\Psi) = ir\left(\partial_r(R)\,e^{i\Psi} + iRe^{i\Psi}\partial_r(\Psi)\right)$$

Cancel out $e^{i\Psi}$.

$$\partial_\theta(R) + iR\partial_\theta(\Psi) = ir\,(\partial_r(R) + iR\partial_r(\Psi))$$

$$\partial_\theta(R) + iR\partial_\theta(\Psi) = ir\,\partial_r(R) - rR\partial_r(\Psi)$$

Separate out real and imaginary parts.

$$\partial_\theta(R) = -\,rR\partial_r(\Psi))$$

$$R\partial_\theta(\Psi) = r\,\partial_r(R)$$

Done.