Needham, Chapter 3, Ex. 4

Palmer, March 22, 2021, 1:50 pm PDT

Out[434]=

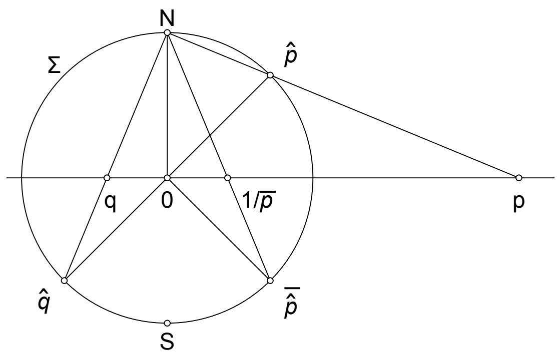

Figure 1. $q = -1/\overline{p}$

4 Deduce (22), p. 148 directly from (17), p. 143.

(22) If $\hat{p}$ and $\hat{q}$ are antipodal points of Σ, then their stereographic projections p and q are related by the following formula:

$$q = -(1/\overline{p}).$$

(17) Inversion of $\mathbb{C}$ in the unit circle induces a reflection of the Riemann sphere in its equatorial plane, $\mathbb{C}$.

Figure 1 is based on [21b], p. 142, which shows a vertical cross-section of Σ through N and and a point in $\mathbb{C}$. For this exercise pick any point $\hat{p}$ in Σ at its intersection with the cross-section, and plot the antipode $\hat{q}$ of $\hat{p}$. Points p and q are obtained by projection from $\hat{p}$ and $\hat{q}$ to $\mathbb{C}$. Point $(1/\overline{p})$ is obtained by inversion of $\mathbb{C}$ in the unit circle. Point $\overline{\hat{p}}$ is the reflection of $\hat{p}$ obtained by reflection of the Riemann sphere in $\mathbb{C}$ induced by inversion of $\mathbb{C}$ in the unit circle. Points $\hat{q}$ and $\overline{\hat{p}}$ are symmetrical across the Z axis by the symmetry of the reflection of $\hat{p}$ and $\overline{\hat{p}}$ in $\mathbb{C}$ and the consequent symmetry of the projection from $\overline{\hat{p}}$ to $1/\overline{p}$ with the projection from $\hat{q}$ to q. The triangle $qN(1/\overline{p})$ must be isosceles. Therefore, it is apparent that q is the reflection of $(1/\overline{p})$ in the Z axis and that $q = -(1/\overline{p})$.