Needham, Ch. 3, Ex. 2, p. 181.
G. Palmer, March 10, 2021, 10 am PDST

Out[1479]=

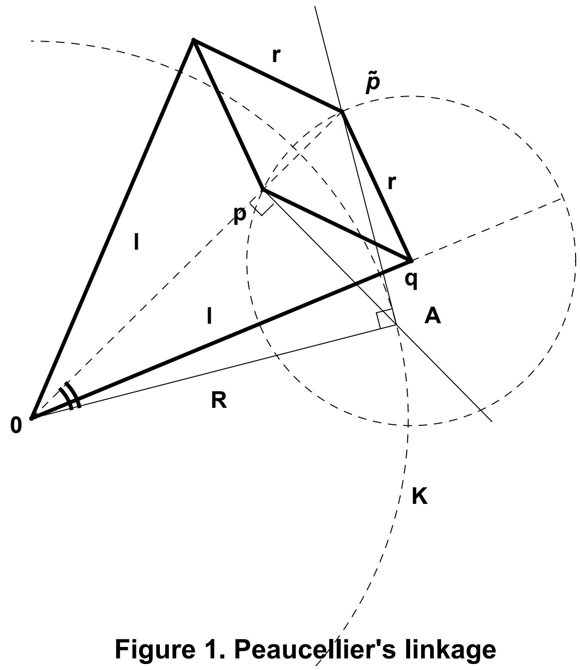

Figure 1. Peaucellier's linkage

2. In 1864 a French officer named Peaucellier caused a sensation by discovering a simple mechanism (Peaucellier's linkage) for transforming linear motion (say of a piston) into circular motion (say of a wheel). The figure below shows six rods hinged at the white dots, and anchored at o. Two of the rods have length l, and the other four have length r. With the assistance of the dashed circle, show that $\tilde{p} = I_K(p)$*, where K is the circle of radius* $\sqrt{l^2 - r^2}$ *centred at o. Construct this mechanism—perhaps using strips of fairly stiff cardboard for rods, and drawing pins for hinges—and use it to verify properties of inversion. In particular, try moving p along a line.*

To prove the inversion of p and $\tilde{p}$ in the circle K, first plot an arbitrary position of the linkage. Then plot a tangent from $\tilde{p}$ to K (at A) and an orthogonal line at p to A. The result is two similar right triangles that share the angle marked with the double thick arcs and the side R. The construction is the same as that in Exercise 1 (i), so $|p|/R = R/|\tilde{p}|$, proving the inversion in the circle.

This proves the inversion, but it does not make use of Needham's instruction "With the assistance of the dashed circle...". Vasco's answer draws the points p and $\tilde{p}$ together at K and plots a tangent there to show that the dashed circle is orthogonal to K. Then it follows from the information on p. 129 that p and $\tilde{p}$ are inverse points in K.

The model in Figure 1 can be animated if one has a Mathematica product in which to run the program.