Needham, Ch. 2, Ex. 40, p. 121.
G. Palmer, February 14, 2021, 7:40 am

Out[•]=

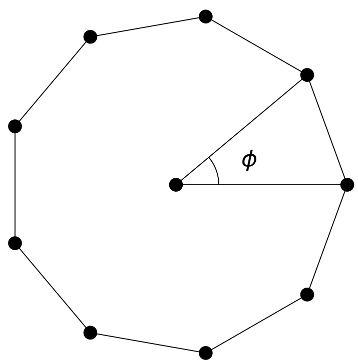

Figure 1. Regular n-gon

40. Here is another simple way of deriving (34). If the vertices of the origin centred regular n-gon are rotated by ϕ, then their centroid Z rotates with them to $e^{i\phi}$ Z. By choosing ϕ = (2Pi/n), deduce that Z = 0.

The answer seems at first to be self-evident, but as Needham pointed out on p. 104, bottom, the answer is not obvious when n is odd. On page 103 (31), we find *If Z is the centroid of* $\{z_j\}$*, then the centroid of* $\{a z_j + b\}$ *is* $aZ + b$.

Let $a = e^{i2\pi/n}$. Since the n-gon is origin-centred, b = 0. Then the centroid of the new n-gon is

$$aZ+b = e^{i2\pi/n} Z + 0 = e^{i2\pi/n} Z$$

but in general, since the origin, and only the origin, is unchanged by rotation by 2π/n, it follows that $e^{i2\pi/n} Z = Z \Longrightarrow Z = 0$. It also follows that if we translate the origin-centred n-gon by b, the new centre will be Z + b = b, which is the new centroid of its vertices. (As Vasco pointed out, see below, the regular n-gon of this exercise itself remains unchanged. But if one marked any one of the vertices, it would be seen to rotate about the origin by 2π/n.)

Vasco has "Rotating the n-gon through (2π/n) leaves it unchanged and so its centroid will also be unchanged, and we can write

$$e^{i2Pi/n} Z = Z \text{ or}$$

$$\left(e^{i2Pi/n} - 1\right) Z = 0$$

Since $\left(e^{i2Pi/n} - 1\right) \neq 0$, it follows that Z = 0 and so we obtain result 34 on page 104 of the book."