

Needham, Chapter 2, Exercise 28, p. 118.

G. Palmer, November 23, 2020, 2:30 pm PDST

28 What value does z^i take at $z = -1$ if we start with the principal value at $z = 1$ (i.e. $1^i = 1$), and then let z travel one and a half revolutions clockwise round the origin?

Rewrite in exponential form. A traversal of one and a half revolutions clockwise is accomplished by multiplying by $e^{-i3\pi}$. Now $z = e^{-i3\pi}$, so

$$z^i = (e^{-i3\pi})^i = e^{3\pi} = 12,391.6 \quad !!$$

Alternatively, we can use the formula presented on p. 102 and derived in the notes for Chapter 2.

$$e^{i2\pi n a} e^{-2\pi n b} [z^i] = e^{-2\pi n},$$

because $z^{a+ib} = z^i = 1$, $a = 0$, $b = 1$,

$$e^{i2\pi n a} e^{-2\pi n b} [z^i] = e^{-2\pi(-3/2)} \quad (\text{because } n = -3/2)$$

$$= e^{3\pi}$$