Discussion of Figures from Needham, Chapter 12, pp. 540-546.

G. Palmer,

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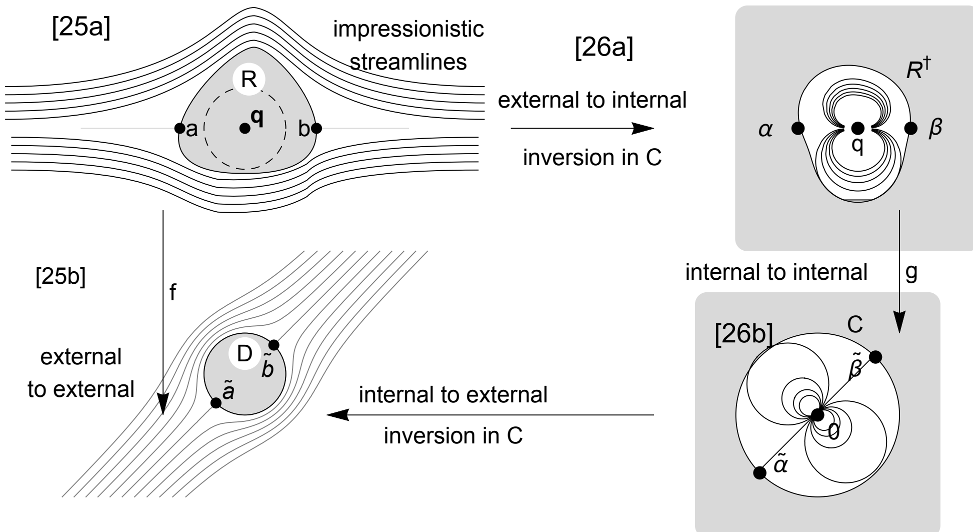
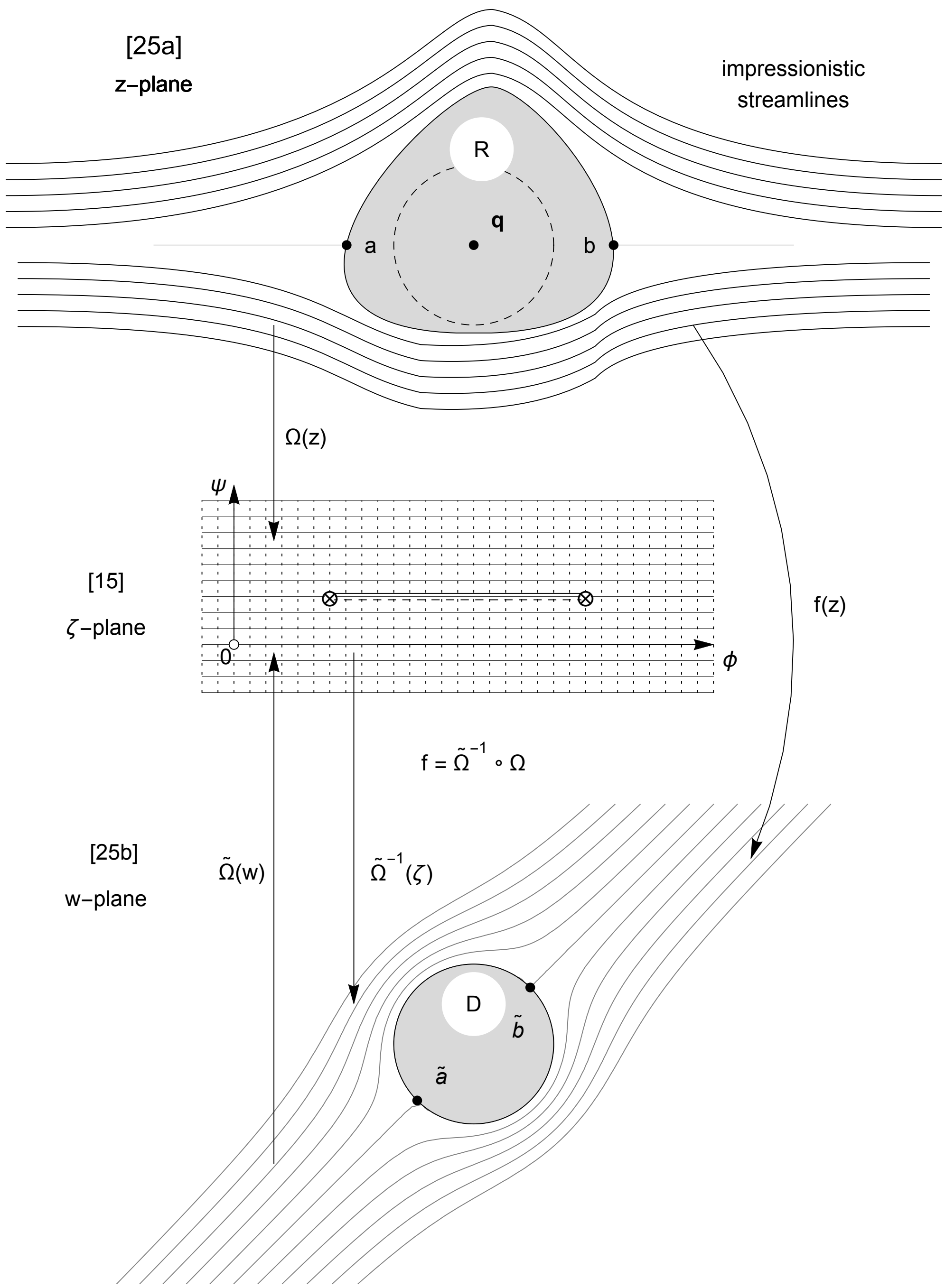
Out[4221]=



The facsimiles to Needham's figures [25] and [26] are imperfect, but perhaps sufficient to our purpose of illustrating Needham's explanation. In [25a], the boundary and lines of flow were created as Bezier curves by trial and error, so perhaps one can take them to be empirically measured flows. A unit disc has been added to provide perspective. The BezierCurve function uses a list of control points to create a curve. This same list of control points can be passed to a function called BezierFunction which creates a function, say F, that draws the curves under ParametricPlot[F(t), {t, min, max}]. The points used in ParametricPlot were extracted from the plot object and subjected to complex inversion in the unit circle: z $\longrightarrow 1/\bar{z}$. Under inversion in the unit circle, here centred at q = 0, the streamlines in [25a] become the dipole in [26a]. If the streamlines of [25a] could be greatly extended, the dipole would close near the center. Under inversion in the unit circle, the boundary of R becomes the boundary of $R^\dagger$ in [26a]. The function f from [25a] to [25b] and the function g from [26a] to [26b] (? f = g) are unspecified and difficult to obtain. One can obtain [25b] from [26b] by inversion of the dipole points in C. All of this assumes that the velocity of the flow is adjusted so that the number of k-cells abutting the boundary between stagnation points a and b, $\tilde{a}$ and $\tilde{b}$, α and β, and $\tilde{\alpha}$ and $\tilde{\beta}$ is equal.

Out[4637]=



The figure shows why $f = \tilde{\Omega}^{-1} \circ \Omega$. Since the letter w is used for f(z) to be consistent with [25], [15] is labeled the ζ-plane.