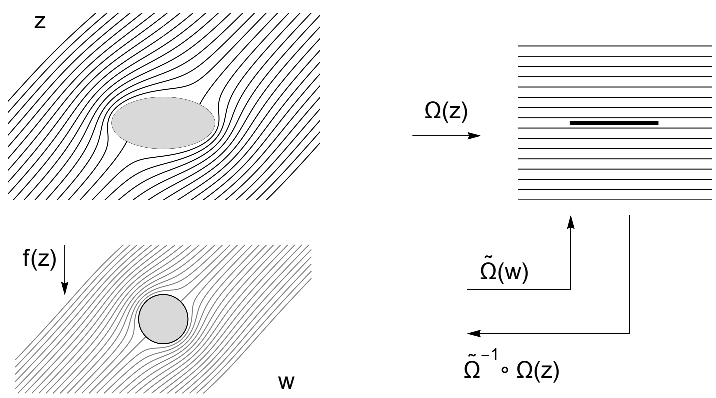
Notes on Chapter 12, Section VI, re. Figure [25], and also [24] from Section V.
April 25, 2019, 8:50 am PST

Sorting out z, w, f, Ω(z), and $\tilde{\Omega}$(w).



The general framework for VI

We are concerned mainly with transforming lines of flow outside and apart from whatever obstacles may have shaped them. The figure does not exactly represent any figure in Needham. For example, in [25] distant lines of the z-plane (top) flow eastward rather than north-eastward as we have plotted them. In [24] the order of the disc and ellipse is reversed under f(z). The figure is intended to show that we can begin with any disturbed uniform flow and regard that as the z-plane. A transformation Ω(z) sends the ψ-lines to horizontal lines. Alternatively, an analytic function w = f(z) sends the ψ-lines to a new disturbed flow in the w-plane shown on the lower left. A transformation $\tilde{\Omega}$(w) sends the ψ-lines from the w-plane to the same set of horizontal lines on the right. I don't quite understand the mechanics of guaranteeing that they are the same, but Needham expends some effort on demonstrating that they are, i.e. that $\tilde{\Omega}(w) = \Omega(z)$, as on p. 543. Then, by this equation, $\tilde{\Omega}^{-1} \circ \Omega(z)$ sends the horizontal lines back to the w-plane, so $f = \tilde{\Omega}^{-1} \circ \Omega(z)$.

If this scheme is correct, then perhaps we can apply it to [24] in section V by letting $w = f^{-1}(z)$ rather than f(z) in the above figure and rearranging the Ω's accordingly. In fact, it does work, as Needham has "$\tilde{\Omega}(w) = f^{-1}(w) + \left[1 \big/ f^{-1}(w)\right]$ is the flow round an obstacle whose boundary curve B is f(C)" (p. 540, ¶2). This is merely another way of writing $\tilde{\Omega}(w) = \Omega(z)$.

$$\tilde{\Omega}(w) = f^{-1}(w) + \left[1 \big/ f^{-1}(w)\right] = z + 1/z = \Omega(z).$$

I think all of this requires that we stipulate that $\tilde{\Omega}(w) = \Omega(z)$.