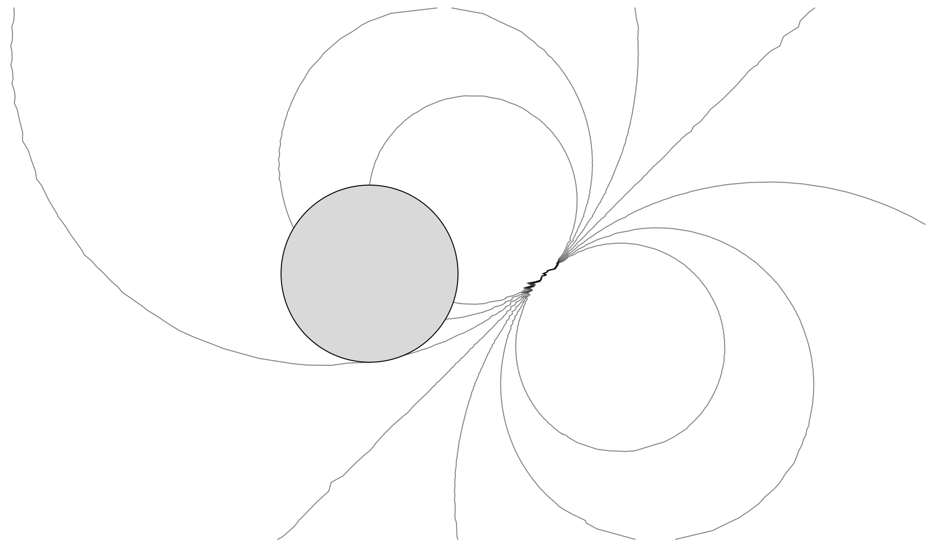
**Notes on Chapter 12, Section V, Flow Around an Obstacle, 2 [16], dual of [12]**
**April 4, 2019, 12:20 pm PST**

**Figure 7. $\Omega_u$ = (1+i)/(z−2)**



**P. 532**. (13) Figure 7

$$\Omega_u = \frac{1+i}{z-2}$$

$$\phi_u = \mathrm{Re}[\Omega_u]$$

$$\psi_u = \mathrm{Im}[\Omega_u]$$

**Figure 8. $\Omega_u$ = log(z−2−i) , undisturbed**

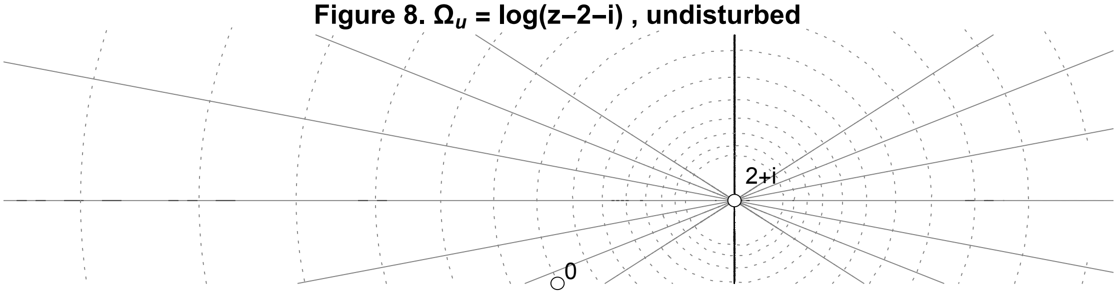
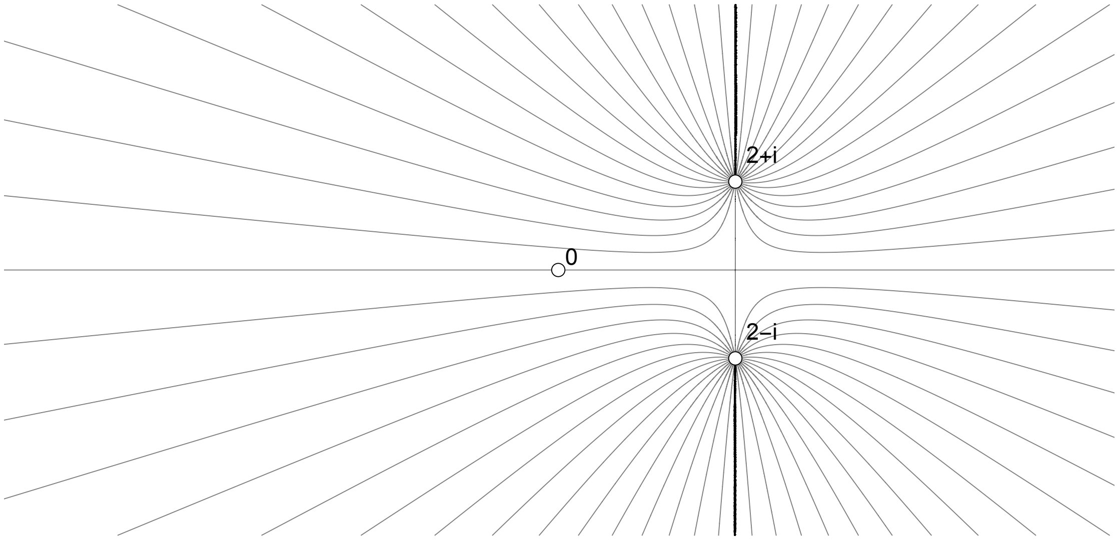


Figure 9. $\Omega_u$ = log(z–2–i) + log(z–2+i), disturbed




**P. 532**. $\Omega_u(z) = \log(z-2-i)$, **"disturbed" and "undisturbed"**

On first reading of this part of the section, I formed the mistaken impression that the equation $\Omega_u(z) = \log(z-2-i)$ would result in an image like [18]. In fact, it resulted in Figure 8. This is the "undisturbed" flow. Figure [18], p. 533, is actually just the upper half plane *after the mirror image has been added*, as in [19], p. 534. It is the UHP of the disturbed flow. To obtain the disturbed flow of [18], [19], or Figure 9, one must add the reflection across the real line.

$$\overline{H} = \frac{1}{\bar{z}-(2+i)}$$

$$\text{H} = \frac{1}{z-2-i}$$

$$\Omega_u(z) = \int \text{H dz} = \log(z-2-i)$$

$$= (1/2)\ \ln[(x-2)^2 + (y-1)^2] + \text{i arctan((y-1)/(x-2))}$$

$\Omega_d$ = log(z-2-i) + log(z-2+i)    Adds reflection across real line.

$\Omega_o$ = log(z-2+i) = (1/2) $\ln[(x-2)^2 + (y+1)^2]$ + i arctan((y+1)/(x-2))

$\Omega_d = \Omega_u + \Omega_o$     Disturbed = Undisturbed + Obstacle

Since the ln|Z| is real, we use the real part $\psi_d$ of $\Omega_d$ to obtain the "disturbed" streamlines.

$\psi_d$ = arctan((y-1)/(x-2)) + arctan((y+1)/(x-2))