

Needham, Chapter 11, Exercise 13, p. 507.
 G. Palmer, September 3, 2018, 8:15 am PDT

13 Continuing the previous question, what would the new complex potential be if fluid were flowing down the channel with speed v prior to the insertion of the dipole? Check your answer by using a computer to draw the streamlines.

The problem is analogous to example (2) on p. 502, except that we are given the speed of flow v , so $\Omega = vz$ for the uniform flow.

Find the new complex potential Ω .

$$\text{Let } \Omega_A = vz. \text{ Let } \Omega_B = \frac{1}{2} \left[\frac{e^z + 1}{e^z - 1} \right]$$

$$\begin{aligned} \Omega &= \Omega_A + \Omega_B = vz + \frac{1}{2} \left[\frac{e^z + 1}{e^z - 1} \right] \\ &= vx + ivy + \frac{1}{2} \frac{e^{2x} - 2i e^x \sin(y) - 1}{e^{2x} - 2e^x \cos(y) + 1} \quad (\Omega_B \text{ was calculated in exercise 12.}) \end{aligned}$$

The streamlines are plotted as contours on ψ , the imaginary part of Ω (Figures 1, 2).

$$\psi(x, y) = \psi_A + \psi_B = vy - \frac{e^x \sin(y)}{e^{2x} - 2e^x \cos(y) + 1} \quad (\psi_B \text{ was calculated in exercise 12.})$$

Figure 1.

$$\psi = vy - (e^x \sin y)/(e^{2x} - 2e^x \cos y + 1)$$

$$v = 2, k = 1$$

$$-7 \leq \psi \leq 7$$

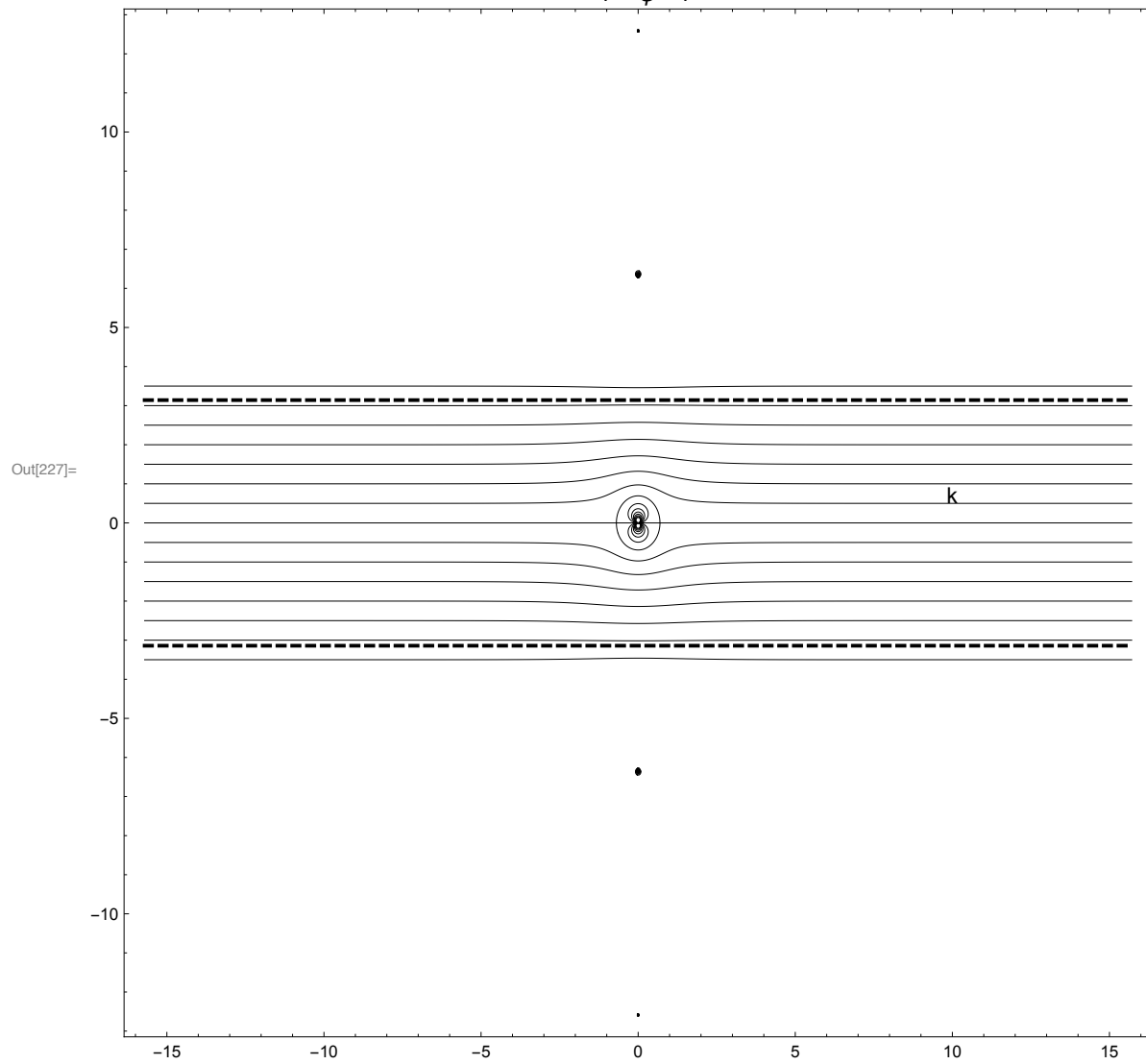


Figure 2.

$$\psi = vy - (e^x \sin y)/(e^{2x} - 2e^x \cos y + 1)$$

$$v = 2, k = 1$$

$$-7 \leq \psi \leq 7$$

