

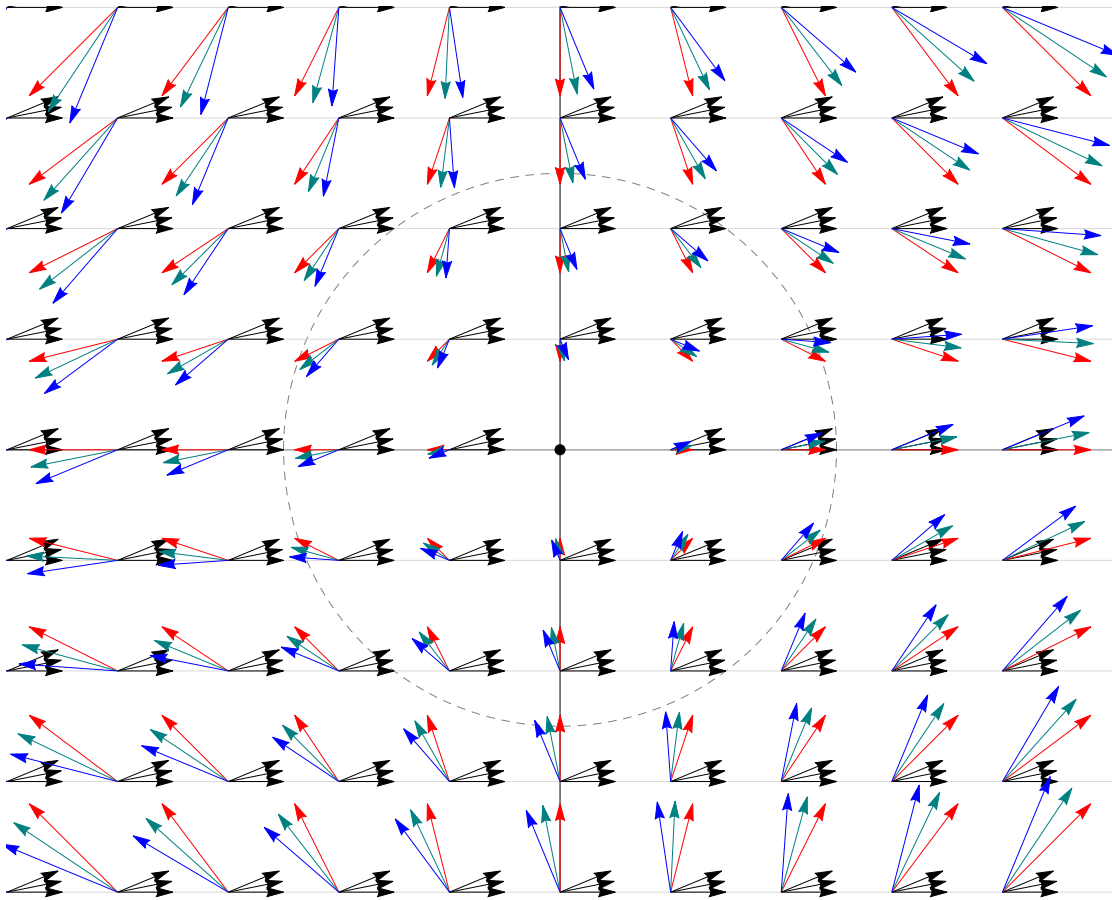


Figure 1: Field Vectors of $C\bar{z}$

10 Reconsider the term $C\bar{z}$ in the local decomposition (17) of a general vector field. See [11].

(i) Show that the visual appearance of the vector field $C\bar{z}$ is essentially independent of the value of C . More precisely, show that if $C = e^{i\phi}$ then increasing ϕ merely causes the entire picture of the vector field $C\bar{z}$ to rotate, in fact exactly half as fast as $e^{i\phi}$ rotates.

Rewrite of question:

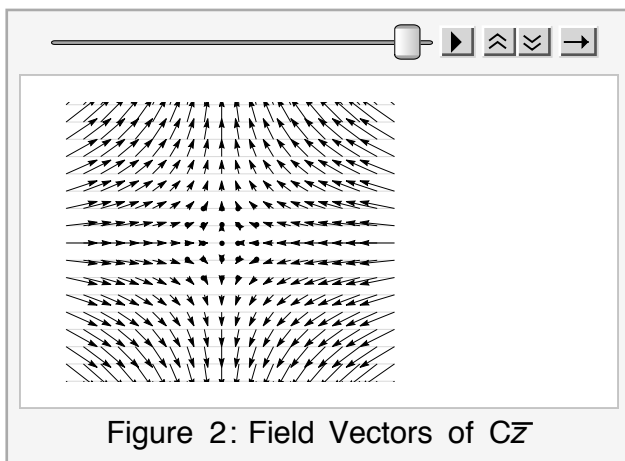
(i) Show that the visual appearance of the vector field $C\bar{z}$ is essentially independent of the value of C . More precisely, if $C = e^{i\phi}$ then increasing ϕ causes the all the vectors of the vector field $C\bar{z}$ to rotate in place by ϕ radians. Show that the same effect can be achieved by rotating the complex plane by $\phi/2$ while the field vectors remain at fixed angles to z .

The black arrows in Figure 1 represent three vector fields $C = .2 e^{i\phi}$ at $\phi = n\pi/16$, where $n = \{0, 1,$

2}. The red arrows represent the vector field $\bar{z}$. The pattern of the red arrows is the same as that on the lower right in Figure [11], p. 487. The blue-green and blue arrows represent two vector fields of $C\bar{z}$ in order of increasing ϕ .

Figure 1 appears to contradict the proposition that “the visual appearance of the vector field $C\bar{z}$ is essentially independent of the value of C .” The phrase “visual appearance” is somewhat vague, but perhaps refers to the length and angle of the field vectors. In Figure 1, length of the field vectors is constant at z , but ϕ varies causing the angle of $C\bar{z}$ to vary at apparently the *same* rate as $e^{i\phi}$. It is easy to see that $\arg(C\bar{z}) = \arg(\bar{z}) + \arg(C)$, as one would expect for multiplication. Hence, the statement that $C\bar{z}$ rotates “exactly half as fast as $e^{i\phi}$ rotates” appears to refer to some other construction of the figure. I have suggested a different construction in the rewrite of the question.

If we assume that the angles between the field vectors and the rays are fixed, and we rotate the complex plane by $\phi/2$, then we find that the field vectors have rotated by $\phi/2$ when the tail of each field vector is translated to 0. To see this, consider the field vectors illustrated in [11d] (i.e. the bottom right figure). Focus on just those positive vectors on the real line. Imagine that they are rigidly attached to the real line at angle 0. Rotate the complex plane by $\pi/2$, so that the vectors now appear to lie on the y axis and point north. To obtain this same configuration by applying C to the vector field $\bar{z}$ on the y axis would require $\phi = \pi$. This is equivalent to allowing the field vectors to rotate independently of the y axis and rotating them by π . The effect is the same for the vector fields on all the rays from the origin.



(ii) To make the result vivid, create a computer animation of the vector field $e^{i\phi} \bar{z}$ as ϕ increases from 0 to π .

See figure 2. The figure can not be animated in this document, but it can be compared with the unrotated image in Figure [11], lower right. The field vectors in Figure 2 have undergone rotation under $\phi = \pi$. The animation is available as a QuickTime MOV file or as a downloadable AVI file which opens and converts to a MOV file. One would prefer a larger image, but even at this size the AVI file is over 10 MB.

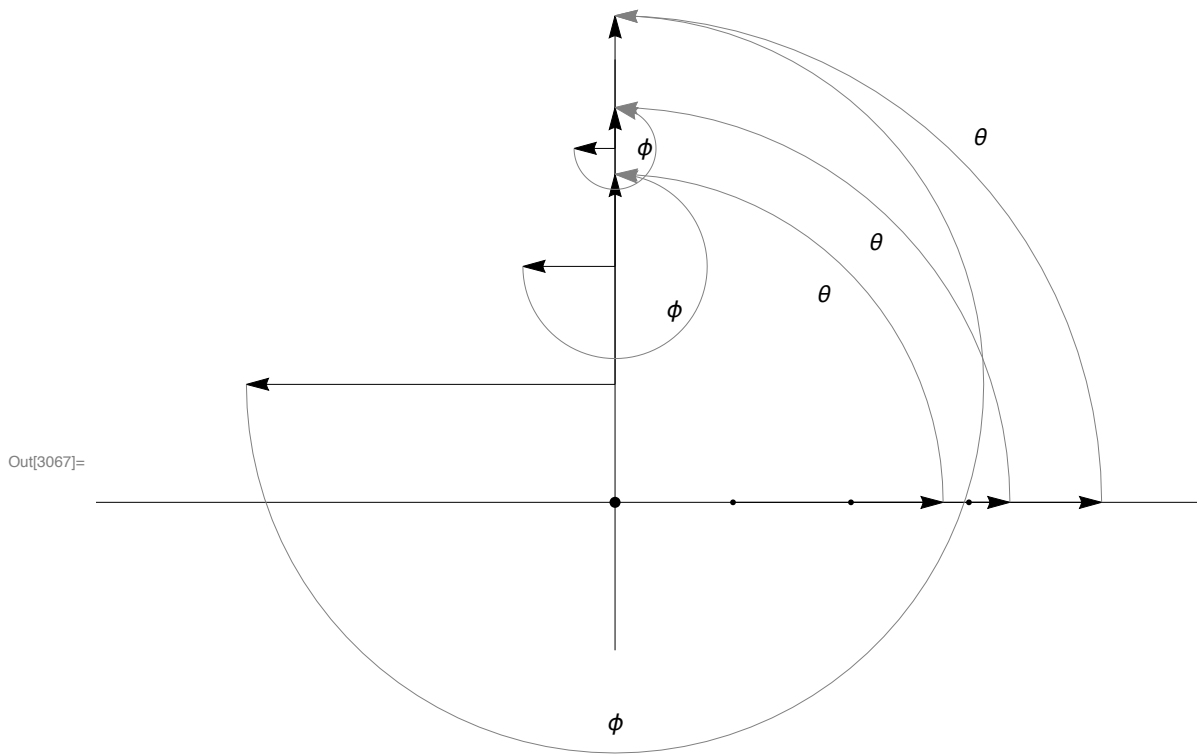


Figure 3: Field Vectors of Cz^{-n} , $\phi = 3\pi/2$, $\theta = \phi/(n+1)$, $n = 2$

(iii) More generally, show that if n is an integer and $F(z)$ stands for either $\overline{z}^n$ or z^{-n} , then the vector field of $e^{i\phi} F$ is obtained by rotating the vector field of F through $\phi/(n+1)$. [Note that the $n = -1$ fields (including sources and vortices) are exceptional.]

Suggested rewrite of question

(iii) More generally, show that if n is an integer and $F(z)$ stands for either $\overline{z}^n$ or z^{-n} , then the vector field of $e^{i\phi} F$ may be obtained by rotating the complex plane through $\phi/(n+1)$.

Let $F = z^{-n} = 1/z^n = 1/r e^{in\theta}$

$$e^{i\phi} F = \frac{e^{i\phi}}{r e^{in\theta}} = \frac{1}{r} e^{i(\phi - n\theta)}$$

It appears that the vector field of F is rotated through ϕ rather than $\phi/(n+1)$, but as in part (i), we can obtain the same result as multiplying the field by $C = e^{i\phi}$ by rotating the complex plane, including fixed field vectors, by $\phi/(n+1)$. See Figures 3, 4.

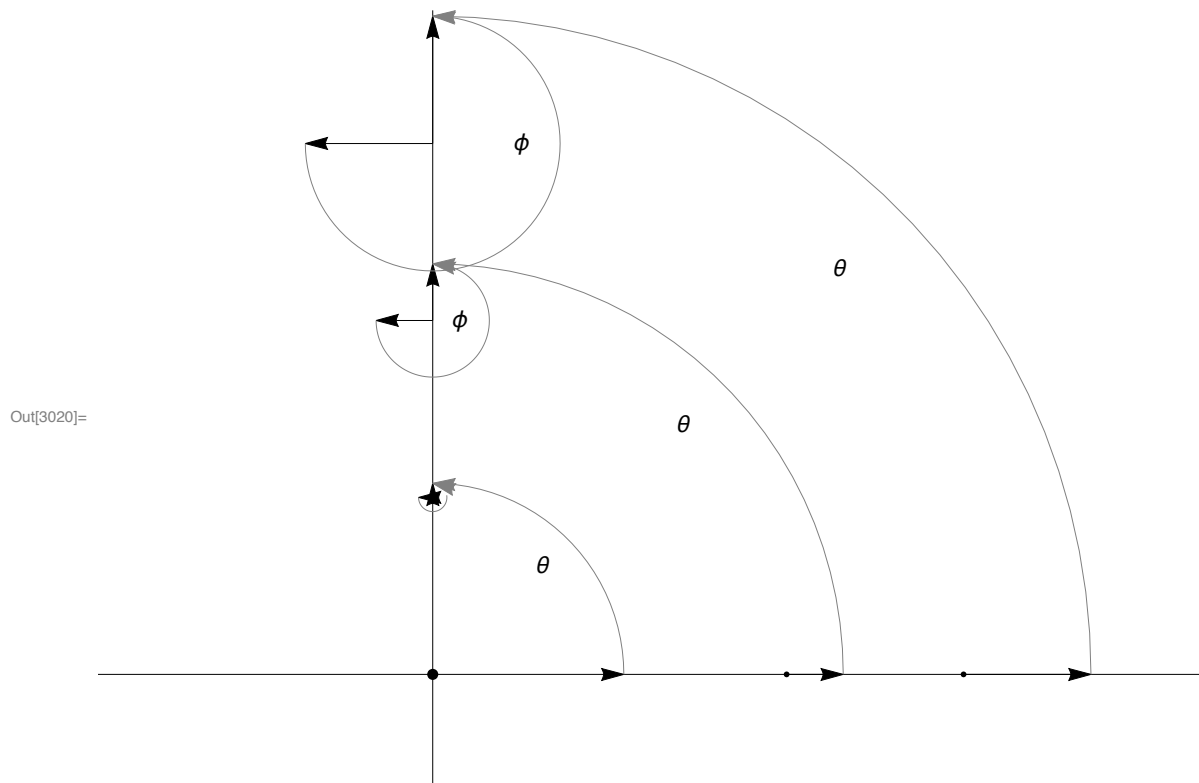


Figure 4: Field Vectors of $C\overline{z}^n$, $\phi = 3\pi/2$, $\theta = \phi/(n + 1)$, $n = 2$