

Needham, Chapter 10, Exercise 9, p. 469.

G. Palmer, February 13, 2018, 9:40 am, PT

9 Continuing from the previous exercise, apply the result as follows:

(i) *By taking $g = f$, deduce that $(f \circ f)$ has a fixed point.*

We know from Exercise 8 that at least one of f , g , or $f \circ g$ has a fixed point. Since $g = f$, if g or f has a fixed point, then both have a fixed point. If f has a fixed point, then

$$(f \circ f)(p) = f(f(p)) = f(p) = p$$

showing that $(f \circ f)(p)$ has a fixed point.

If one of these does not have a fixed point, then neither has a fixed point, in which case $(f \circ f)$ has a fixed point, because one of the three must have a fixed point.

(ii) *By taking $g = (\text{antipodal mapping}) [A]$, deduce that either f has a fixed point, or f maps some point to its antipodal point.*

We are given that $g(p) = A(p)$, so g is not a fixed point. Then the essential fixed point might be either f or $f \circ g$. If f lacks a fixed point, then $f \circ g$ must have a fixed point, in which case

f must be the antipodal mapping $f(p) = A(p)$:

$$(f \circ g)(p) = f(g(p)) = A(A(p)) = p$$

QED