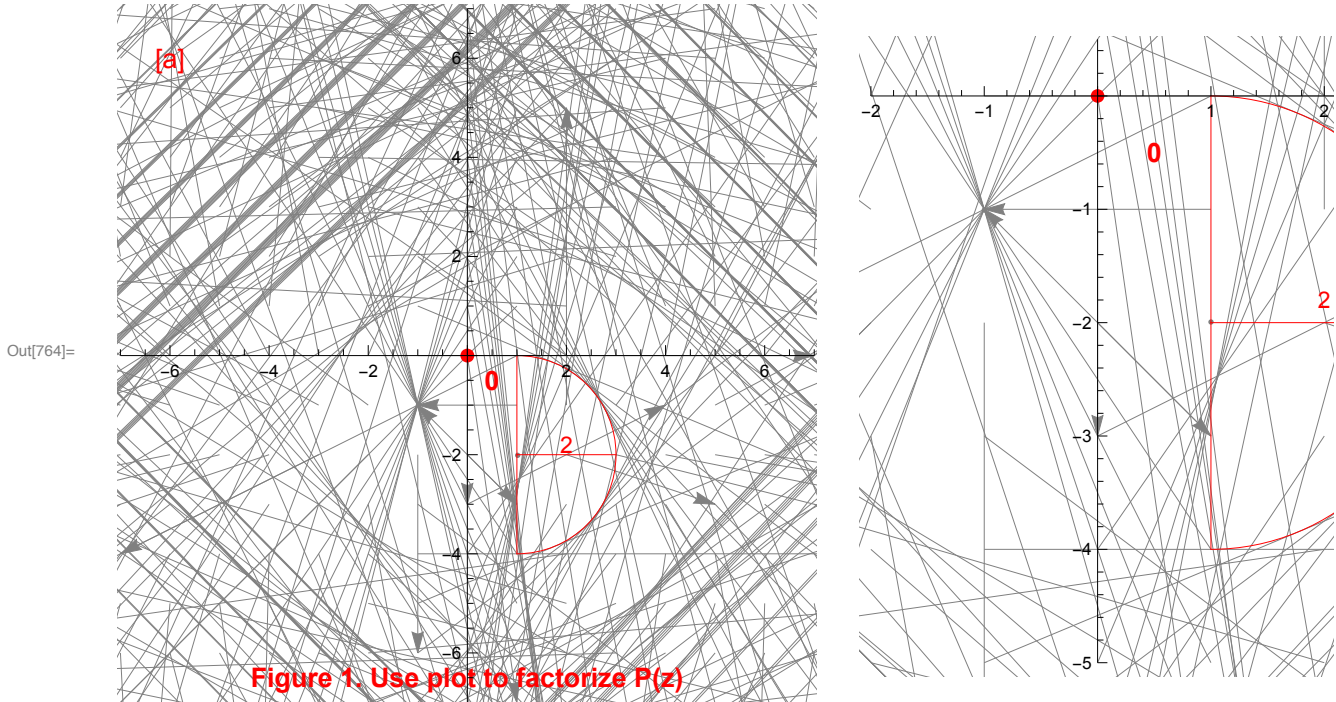




2 Use a computer to draw the vector field of

$$P(z) = z^3 + (-1 + 5i)z^2 + (-9 - 2i)z + (1 - 7i).$$

Use this picture to factorize $P(z)$, and check your answer by multiplying out the brackets.

Judging from $P(z)$ as written, there are no poles, so we need only be concerned with roots. If a root has order 1, vectors appear to emanate directly from the root or to point towards the root, because a vector V based on z has the same direction as z . For example, a factor $(z - 2)$ would have direction $\arg(z)$ and emanate from the point 2. Then if $z = -1$, the field vector would point from 2 to $(-1 - i)$. In Figure 1, vectors both emanate from and point towards the point $(-1 - i)$. For this root, a factor of $(z + 1 + i)$ seems likely.

Two more roots are needed. One can discern and plot a half circle centered at $(1 - 2i)$ with a radius of 2. This circle is analogous to the circle streamline in exercise 1. The vectors on this circle appear to be order 2 and tangent to the circle. And this circle is tangent to the real line, but at 1 rather than at the origin as it was for z^2 . But both are tangent to the real line at the real value of the root. The streamline circle of z^2 was not centred at the root 0. If we choose a factor $(z - (1 - 2i))$ of order 2, the construction of the streamline circle will differ in this respect. Nevertheless, this factor seems to

work.

$$P(z) = (z + (1 + i)) (z - (1 - 2i))^2$$

The same answer is obtained by multiplying out the brackets.

```
In[822]:= (* work sheet *)

Clear[p, z]

p = z^3 + (-1 + 5 I) z^2 + (-9 - 2 I) z + (1 - 7 I);

simpP = Simplify @ p;

Print["factorization check, P(z) = ", simpP]

factorization check, P(z) = (z - (1 - 2 i))^2 (z + (1 + i))
```