Needham, Chapter 1.
G. Palmer, February 24, 2021, 10:20 am, PST

p. 7, line - 3

or one could write

$$\frac{(1+i)^5}{\left(\sqrt{3}+i\right)^2} = \frac{\sqrt{2}^5 \angle 5\tan^{-1}(1)}{\sqrt{3+1}^2 \angle 2\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)} = \sqrt{2}\angle[(5\ \texttt{tan}^{-1}\ (1) - 2\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)]$$

$$= \sqrt{2}\angle[5(\pi/4) - 2(\pi/6)] = \sqrt{2}\angle(15\text{-}4)\pi/12 = \sqrt{2}\angle(-\pi/12)$$

Noting that $\angle\theta \equiv e^{i\theta}$, one could write

$$\frac{(1+i)^5}{\left(\sqrt{3}+i\right)^2} = \frac{\sqrt{2}^5\, e^{i5\tan^{-1}(1)}}{\sqrt{3+1}^2\, e^{i2\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)}} = \frac{4\sqrt{2}\, e^{i5\pi/4}}{4\, e^{i2\pi/6}} = \frac{\sqrt{2}\, e^{i15\pi/12}}{e^{i4\pi/12}}$$

$$= 2\sqrt{2}\ e^{i11\pi/12} = \pm\sqrt{2}\angle(-\pi/12)$$

P. 18 *with a single rotation:* $\mathcal{R}_0^\theta \circ \mathcal{T}_v = \mathcal{R}_p^\theta$. *What is p?*

$$e^{i\theta}(z+v) = e^{i\theta}(z-p) + \mathrm{p}$$

$$e^{i\theta}z + e^{i\theta}v = e^{i\theta}z - e^{i\theta}p + \mathrm{p}$$

$$p\left(1 - e^{i\theta}\right) = e^{i\theta}v$$

$$\mathrm{p} = e^{i\theta}v/\left(1 - e^{i\theta}\right)$$

p. 18 [exercise]

$$\mathcal{R}_a^\theta(\mathrm{z}) = e^{i\theta}(z-a) + \mathrm{a}$$

$$\mathcal{R}_b^\phi = e^{i\phi}(z-b) + \mathrm{b}$$

$$(\mathcal{R}_b^\phi \circ \mathcal{R}_a^\theta)(\mathrm{z}) = e^{i\phi}\left(e^{i\theta}(z-a) + a - b\right) + \mathrm{b}$$

$$= e^{i\phi}\left(e^{i\theta}z - e^{i\theta}a\right) + a - b\right) + \mathrm{b}$$

$$= e^{i(\phi+\theta)}z - e^{i(\phi+\theta)}a + e^{i\phi}a - e^{i\phi}b + \mathrm{b}$$

$$= e^{i(\phi+\theta)}z + \mathrm{v}, \text{ where } \mathrm{v} = -e^{i(\phi+\theta)}a + e^{i\phi}a - e^{i\phi}b + \mathrm{b}$$

$$\text{v} = -e^{i(\phi+\theta)}\, a + e^{\text{i}\phi}\, a - e^{\text{i}\phi}\, b + \text{b}$$

$$= \text{a}e^{\text{i}\phi}\left(1 - e^{\text{i}\theta}\right) + \text{b}\left(1 - e^{\text{i}\phi}\right)$$

What makes this an useful format?

[What should c equal if b = a or ϕ = 0? Check the formula.]

$\mathcal{R}_b^{\phi} \circ \mathcal{R}_a^{\theta} = \mathcal{R}_c^{(\theta+\phi)}$, where $\text{c} = \frac{\text{a}e^{\text{i}\phi}(1-e^{\text{i}\theta})+b(1-e^{\text{i}\phi})}{1-e^{i(\theta+\phi)}}$

If b = a:

$$\text{c} = \frac{\text{a}e^{\text{i}\phi}(1-e^{\text{i}\theta})+a(1-e^{\text{i}\phi})}{1-e^{i(\theta+\phi)}} = \frac{a(e^{\text{i}\phi}(1-e^{\text{i}\theta})+(1-e^{\text{i}\phi}))}{1-e^{i(\theta+\phi)}}$$

$$= \frac{a(1-e^{i(\theta+\phi)})}{1-e^{i(\theta+\phi)}} = \text{a}$$

If $\phi = 0$

$$\text{c} = \frac{1\,(1-e^{\text{i}\theta})+a(1-1)}{1-e^{\text{i}\theta}} = 1$$

P. 19 explanation of result in [12a] using [15a].

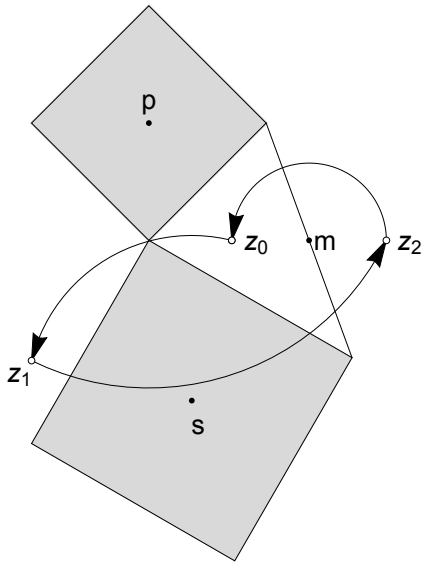


Figure 1. Rotation solution to [12a]

$$\mathcal{M} = \mathcal{R}_m^{\pi} \circ \mathcal{R}_p^{(\pi/2)} \circ \mathcal{R}_s^{(\pi/2)} = \mathcal{E}$$

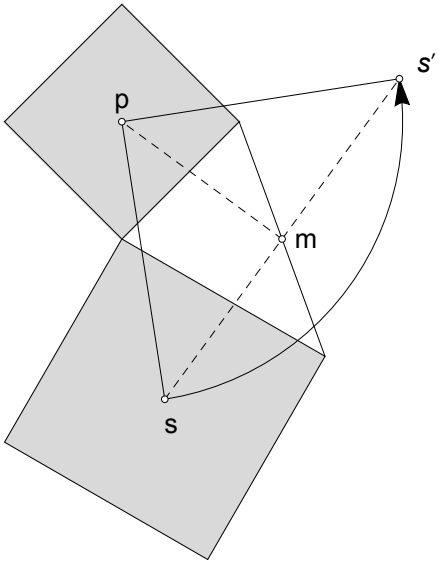


Figure 2. Rotation solution to [12a], Pt. 2

$s' = \mathcal{R}_p^{(\pi/2)}(\mathrm{s}) = \mathcal{E}$

P. 27 Illustration of generalized form of Cotes' result.

Out[1356]=

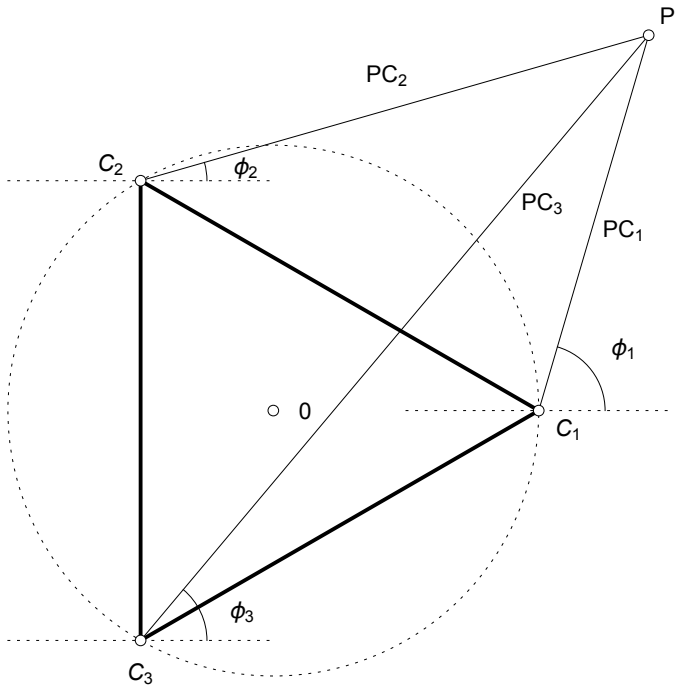


Figure 3. $U_3(P) = PC_1 \cdot PC_2 \cdot PC_3] e^{i\Phi}$, $PC_i = |z - C_i|$