Needham, Chapter 1, Exercise 7, p. 46.
G. Palmer, December 29, 2019, 4:30 pm PST

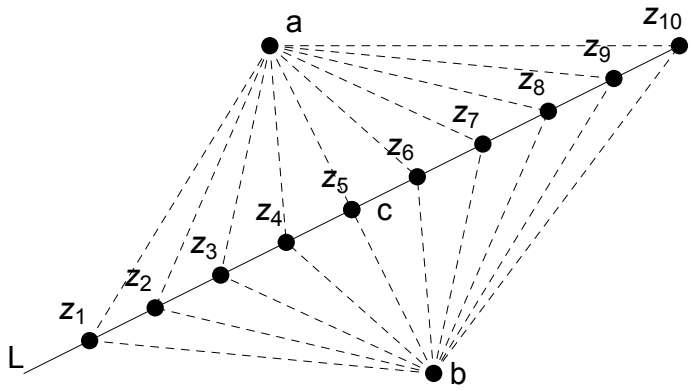


Figure 1. |z−a| = |z−b|

7 Use a picture to show that if a and b are fixed complex numbers then |z-a| = |z-b| is the equation of a line.

In figure 1, the line L is the orthogonal bisector of the line segment [ab] at c. Hence, all the triangles $b - z_i - c$ are reflections of triangles $a - z_i - c$ about L, and they are therefore congruent (with reflection), so |z-a| = |z-b| for every z on L, which makes |z-a| = |z-b| the equation for L.