Needham, Chapter 1, Exercise 6, p. 46.
G. Palmer, December 27, 2019, 6:15 pm PST

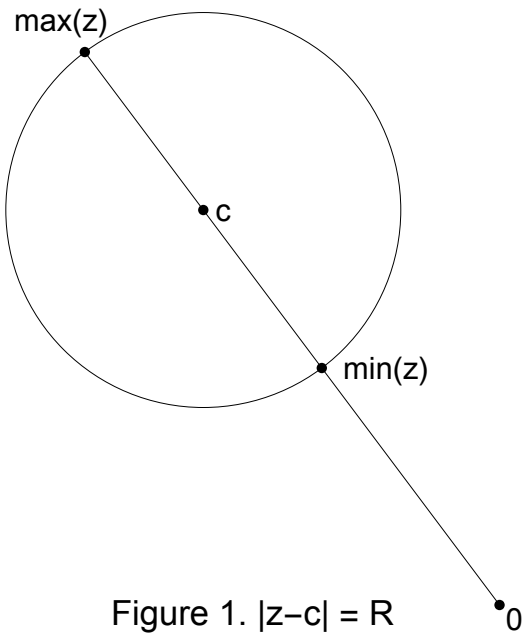


Figure 1. |z−c| = R

6 (i) If c is a fixed complex number, and R is a fixed real number, explain with a picture why |z-c| = R is the equation of a circle.

The equation describes the fixed point centre c and the distance from c to z. Since R and c are fixed, z can only vary by traversing a circle of radius R.

(ii) Given that z satisfies the equation |z + 3 - 4i| = 2, find the minimum and maximum values of |z| and the corresponding positions of z.

c = -3 + 4i

R = 2

min(|z|) = |c| - R = |-3 + 4i| - 2 = $\sqrt{9 + 16}$ - 2 = 3
at z = 3/5 c

max(|z|) = |c| + R = 5 + 2 = 7
at z = 7/5 c