Needham, Chapter 1, Exercise 5, p. 45.
G. Palmer, December 26, 2019, 1:45 pm PST

5 The figure below shows how two similar triangles may be used to constuct the product of two complex numbers. Explain this.

Let S be the length of the segment labeled ab in the figure. Since the triangles are similar, we can write

$$\frac{S}{|b|} = \frac{|a|}{1} \Longrightarrow \text{S} = |\text{a}|\,|\text{b}|$$

This gives us the length of ab. The angle of ab is the sum of ∠a (the angle a01) and ∠b (the angle b01). When we multiply a by b, we multiply their absolute values and add their angles in the exponentials.

$$\text{ab} = S\, e^{\text{i}\angle\text{a}}\, e^{\text{i}\angle\text{b}}$$

$$= |\text{a}|\,|\text{b}|\, e^{i(\angle\text{a} + \angle\text{b})}$$

Hence, the common angle marked with the double arc is added to ∠b to form the angle ab. Given that one side of the "a" triangle is 1, which has corresponding side |b| in the "ab" triangle, to change the length of ab, we could change |b| without changing ∠a. To rotate ab about 0, we can change ∠b without changing |b|.