Needham, Chapter 1, Exercise 44, pp. 54.
G. Palmer, May 16, 2020, 5 pm PST

44 Just as in [13b], a dialative rotation $\mathcal{D}_p^{r,\theta}$ centred at an ordinary point p may be performed by translating p to the origin, doing $\mathcal{D}_0^{r,\theta}$, then translating 0 back to p. Representing these transformations as complex functions, show that

$$\mathcal{D}_p^{r,\theta}(\mathrm{z}) = re^{i\theta} z + v, \text{ where } v = p\left(1 - re^{i\theta}\right).$$

Conversely, if v is given, deduce that

$$\mathcal{T}_v \circ \mathcal{D}_0^{r,\theta} = \mathcal{D}_p^{r,\theta}, \quad \text{where } p = v \Big/ \left(1 - re^{i\theta}\right).$$

$$\mathcal{D}_p^{r,\theta}(\mathrm{z}) = \mathcal{D}_0^{r,\theta}(\mathrm{z} - \mathrm{p}) + \mathrm{p}$$

$$= \mathcal{D}_0^{r,\theta}(\mathrm{z}) - \mathcal{D}_0^{r,\theta}(\mathrm{p}) + \mathrm{p}$$

$$= \mathrm{re}^{\mathrm{i}\theta}\,\mathrm{z} - re^{i\theta}\mathrm{p} + \mathrm{p}$$

$$= \mathrm{re}^{\mathrm{i}\theta}\,\mathrm{z} + p\left(1 - \mathrm{re}^{\mathrm{i}\theta}\right)$$

$$= \mathrm{re}^{\mathrm{i}\theta}\,\mathrm{z} + \mathrm{v}$$

This is what we wanted. Conversely

$$\mathcal{T}_v \circ \mathcal{D}_0^{r,\theta} = \mathrm{re}^{\mathrm{i}\theta}\,\mathrm{z} + \mathrm{v} = \mathrm{re}^{\mathrm{i}\theta}\,\mathrm{z} + p\left(1 - \mathrm{re}^{\mathrm{i}\theta}\right) \quad \text{(from above)}$$

$$= \mathrm{re}^{\mathrm{i}\theta}\,\mathrm{z} + \left[v \Big/ \left(1 - \mathrm{re}^{\mathrm{i}\theta}\right)\right]\left(1 - \mathrm{re}^{\mathrm{i}\theta}\right)$$

$$= \mathrm{re}^{\mathrm{i}\theta}\,\mathrm{z} + \mathrm{v} = \mathcal{D}_p^{r,\theta}(\mathrm{z})$$

This is what we wanted.