Needham, Chapter 1, Exercise 41, pp. 53.
G. Palmer, May 5, 2020, 12:15 PST

41 Let $\tilde{\mathcal{M}}(z)$ be the representation of a general opposite motion as a complex function.

(i) Explain why $\overline{\tilde{\mathcal{M}}(z)}$ is a direct motion, and deduce from (27) that $\tilde{\mathcal{M}}(z) = e^{i\alpha}\overline{z} + w$, for some α and w.

From (25) on p. 36, $\tilde{\mathcal{M}}$(z) = ($\mathcal{M}$ followed by reflection in a line) and $\tilde{\mathcal{M}}(z)$ is an opposite motion. Therefore, $\overline{\tilde{\mathcal{M}}(z)}$ is a direct motion because it reflects $\tilde{\mathcal{M}}$(z) in the x-axis. (27) states that every direct motion can be expressed as a complex function of the form $\mathcal{M}$(z) = $e^{i\theta}\, z$ + v. Then we can write

$$\tilde{\mathcal{M}}(z) = \overline{\mathcal{M}(z)} = \overline{e^{i\theta}\, z + v} = e^{-i\theta}\,\overline{z} + \overline{v}$$

$$= e^{i\alpha}\,\overline{z} + w, \text{ where } \alpha = -\theta, \text{ w} = \overline{v}$$

(ii) Using the previous exercise [and (i)], deduce that every opposite motion is a glide reflection.

Since an opposite motion can be written $\tilde{\mathcal{M}}$(z) = $e^{i\alpha}\overline{z}$ + w, for some α and w, we can see that $e^{i\alpha}\overline{z}$ implies reflection $\mathfrak{R}_L$ in a line L at the angle α with the x-axis followed by a translation $\mathcal{T}_w$. This is the formula for a glide reflection. Hence, every opposite motion must be a glide reflection.