Needham, Chapter 1, Exercise 4, p. 45.
G. Palmer, December 26, 2019, 1:10 pm PST

4 Here is a basic fact about integers that has many uses in number theory: If two integers can be expressed as the sum of two squares, then so can their product. With the understanding that each symbol denotes an integer, this says that if $M = a^2 + b^2$ and $N = c^2 + d^2$, then $MN = p^2 + q^2$. Prove this result by considering $|(a + ib)(c + id)|^2$

$$|(a + ib)(c + id)|^2 = |a + ib|^2 \, |c + id|^2$$

$$= \left(a^2 + b^2\right)\left(c^2 + d^2\right) = MN$$

Let p = (ac-bd), q = (bc+ad).

$$|(a + ib)(c + id)|^2 = |ac+ibc+iad-bd|^2$$

$$= |(ac-bd)+i(bc+ad)|^2$$

$$= (ac - bd)^2 + (bc + ad)^2 = p^2 + q^2 = MN$$