Needham, Chapter 1, Exercise 37, p. 52.
G. Palmer, April 25, 2020, 1:30 pm PST

Out[2217]=

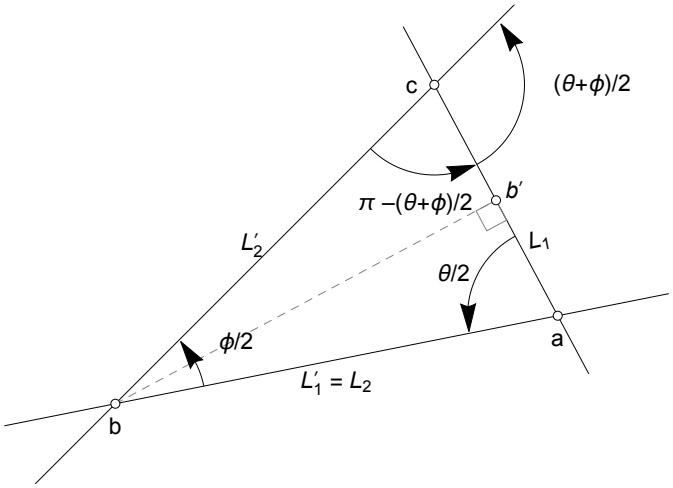


Figure 1. Facsimile of [30b]
with altitude line

37 Deduce (33) directly from [30b]. [Hint: Draw the altitude through b of the triangle abc, and express its length first in terms of sin $\frac{\theta}{2}$*, then in terms of sin* $\frac{(\theta+\phi)}{2}$*.]*

$$\frac{c-b}{a-b} = \left[\frac{\sin\frac{\theta}{2}}{\sin\frac{(\theta+\phi)}{2}}\right] e^{i\phi/2} \qquad (33)$$

This was deduced in my previous answer by using the sine rule and [30b]. Using the hint instead, we plot the altitude line from b to b' (Figure 1) by taking the projection of (b-a) onto (c-a).

$$\left|\frac{b-b'}{a-b}\right| = \sin\frac{\theta}{2}$$

$$\left|\frac{b-b'}{c-b}\right| = \sin\left(\pi - \frac{(\theta+\phi)}{2}\right) = \sin\left(\frac{(\theta+\phi)}{2}\right)$$

$$|b - b'| = |a-b| \sin\frac{\theta}{2} = |c-b| \sin\left(\frac{\theta+\phi}{2}\right)$$

$$\left|\frac{c-b}{a-b}\right| = \frac{\sin\frac{\theta}{2}}{\sin\left(\frac{\theta+\phi}{2}\right)}$$

$\mathrm{Arg}\left(\frac{c-b}{a-b}\right)$ = (arg(c-b) - arg(a-b)) = ϕ / 2 can be read directly from [30b]. Then we can write

$$\frac{c-b}{a-b} = \left|\frac{c-b}{a-b}\right| e^{i\phi/2} = \frac{\sin\frac{\theta}{2}}{\sin\left(\frac{\theta+\phi}{2}\right)} e^{i\phi/2}$$

QED