Needham, Chapter 1, Exercise 27, p. 47.
Palmer, February 5, 2020, 3:10 pm PST

Out[1441]=

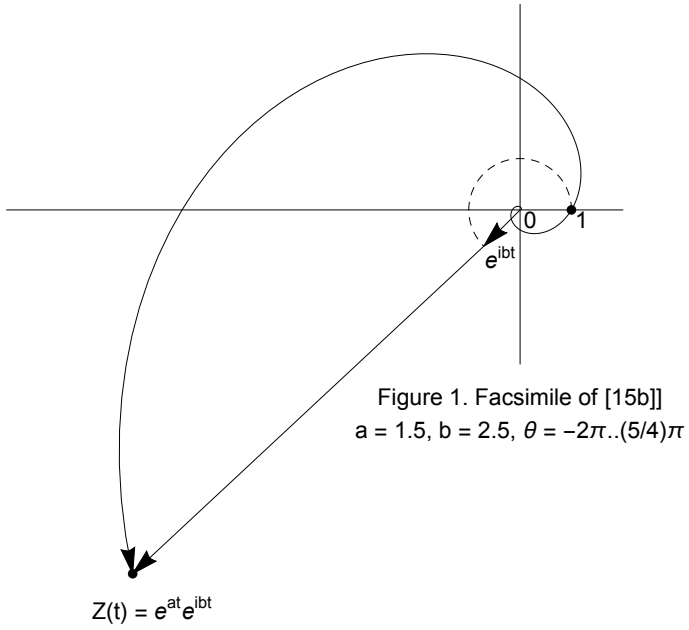


Figure 1. Facsimile of [15b]]
a = 1.5, b = 2.5, $\theta = -2\pi..(5/4)\pi$

27 Show that the polar equation of the spiral Z(t) = e^{at} e^{ibt} in [15b] is r = $e^{(a/b)\,\theta}$.

The dotted circle in [15b] is equivalent to r = $\left| e^{i\theta} \right| = 1$. Hence, θ = bt and t = θ/b. Then write

$$Z(\theta) = e^{a\cdot(\theta/b)}\, e^{i\theta}$$

Then, since $e^{i\theta}$ traverses a circle, it is sufficient to write just the value of r, so r = $e^{(a/b)\,\theta}$.

In Figure 1, note that Z(0) = 1. To actually touch the origin with the spiral inside the unit circle, one would have to begin at $\theta = -\infty$, but that is impractical.