Needham, Chapter 1, Exercise 15, p. 47.
Palmer, January 10, 2020, 8:45 pm PST

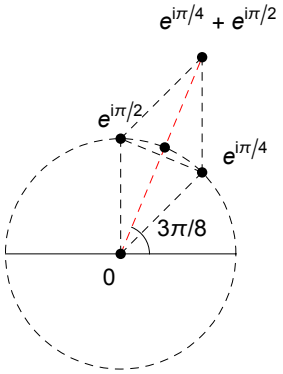



Figure 1. Tan(3π/8) = $1 + \sqrt{2}$

*15 Draw $e^{i\pi/4}$, $e^{i\pi/2}$, and their sum. By expressing each of these numbers in the form (x+iy) deduce that*

$$\tan \frac{3\pi}{8} = 1 + \sqrt{2}$$

$$e^{i\pi/4} = \mathrm{Re}(e^{i\pi/4}) + \mathrm{Im}(e^{i\pi/4})\, i = \sqrt{2}\,/2 + \left(\sqrt{2}\,/2\right) i$$

$$e^{i\pi/2} = \mathrm{Re}(e^{i\pi/2}) + \mathrm{Im}(e^{i\pi/2})\, i = i$$

$$e^{i\pi/4} + e^{i\pi/2} = \sqrt{2}\,/2 + \left(1 + \sqrt{2}\,/2\right) i$$

$$\tan(\arg(e^{i\pi/4} + e^{i\pi/2})) = \mathrm{Im}(...)/\mathrm{Re}(...)$$

$$= \left(1 + \sqrt{2}\,/2\right)/(\sqrt{2}\,/2)$$

$$= 1 + \sqrt{2}$$

Now it appears that we must show that $\tan(\arg(e^{i\pi/4} + e^{i\pi/2})) = \tan\frac{3\pi}{8}$. Then it appears that we must show that $\arg(e^{i\pi/4} + e^{i\pi/2}) = \frac{3\pi}{8}$. Since $e^{i\pi/4}$ and $e^{i\pi/2}$ both have unit length, a rhombus is created by drawing lines from $e^{i\pi/4}$ and $e^{i\pi/2}$ to the point $(e^{i\pi/4} + e^{i\pi/2})$ and to 0. It follows that the diagonal ray bisects a chord from $e^{i\pi/4}$ to $e^{i\pi/2}$ as well as the segment of C between $e^{i\pi/4}$ and $e^{i\pi/2}$ and the angle between $e^{i\pi/4}$ and $e^{i\pi/2}$. Then

$$\arg\left(e^{i\pi/4} + e^{i\pi/2}\right) = \ 1/2\ (\pi/4 + \pi/2) = 3\pi/8$$

Done.