Needham, Chapter 1, Exercise 11, p. 46.
G. Palmer, January 1, 2019, 1:35 pm PST

Out[1511]=

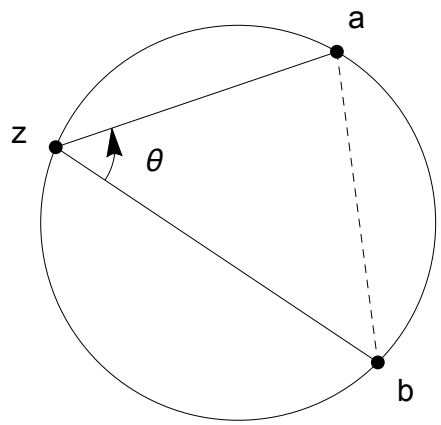

Figure 1. arg((z−a)/(z−b)) = const.

11 Explain geometrically why the locus of z such that

$$arg\left(\frac{z-a}{z-b}\right) = const.$$

is an arc of a certain circle passing through the fixed points a and b.

In figure [28], p. 36, we learn that chord AA′ subtends the same angle θ at every point of the circular arc AM*A′*. Therefore, in Figure 1, chord ab subtends the same angle θ at every point of the circular arc azb, showing that

$$arg\left(\frac{z-a}{z-b}\right) = const.$$

is an arc of a certain circle passing through the fixed points a and b.

θ = arg(a-z) - arg(b-z)

$$= \arg\left(\frac{a-z}{b-z}\right) = \arg\left(\frac{-(z-a)}{-(z-b)}\right) = \arg\left(\frac{z-a}{z-b}\right) = \text{const.}$$