Needham, Chapter 1, Exercise 10, p. 46.
G. Palmer, January 1, 2019, 12:45 pm PST

Out[1135]=

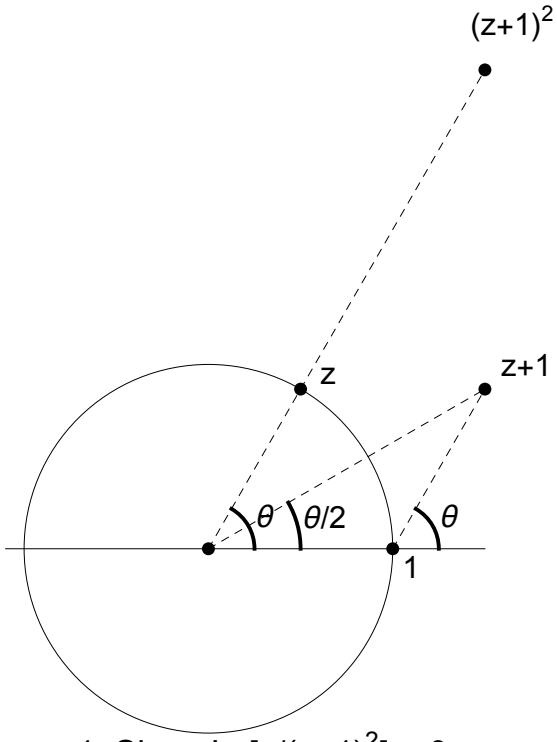


Figure 1. Show Im[z/(z+1)^2] = 0

10 Show geometrically that if |z| = 1 then

$$Im\left[\frac{z}{(z+1)^2}\right] = 0.$$

Figure 1 shows that $\angle (z+1)^2 = \angle z$. This implies that $\arg(\frac{z}{(z+1)^2}) = 0$, in which case it follows that $\mathrm{Im}\left[\frac{z}{(z+1)^2}\right] = 0$.

Apart from the unit circle, what other points satisfy this equation?

Points on the real line satisfy it trivially. Points on a circle |z| = r satisfy the more general equation

$$\mathrm{Im}\left[\frac{z}{(z+r)^2}\right] = 0.$$