Needham, VDGF, Chapter 7, Exercise 29, p. 92.
G. Palmer, September 6, 2022, 4:30 pm PDT

29. Curvature of Hemispherical $\mathbb{H}^2$ *in* $\mathbb{H}^3$*. Use the curvature formula (4.10) to verify that the horizon-centred hemispheres of* $\mathbb{H}^3$ *with metric (6.24) are indeed* $\mathbb{H}^2$*'s with constant negative curvature* $\mathcal{K} = -1$*.*

In the notes for p. 81, we calculated the following. Find A and B as the coefficients of $d\theta$ and $d\phi$.

$$d\hat{s}^2 = \frac{\sin^2\phi\, d\theta^2 + d\phi^2}{\cos^2\phi} = \frac{\sin^2\phi}{\cos^2\phi}\, d\theta^2 + \frac{1}{\cos^2\phi}\, d\phi^2$$

$$A = \tan\phi, \qquad B = \frac{1}{\cos\phi}$$

$$AB = \frac{\tan\phi}{\cos\phi}$$

$$\mathcal{K} = -\frac{\cos\phi}{\tan\phi}\left[\partial_\phi\left(\frac{\partial_\phi \tan\phi}{\frac{1}{\cos\phi}}\right) + \partial_\theta\left(\frac{\partial_\theta\left(\frac{1}{\cos\phi}\right)}{\tan\phi}\right)\right]$$

$$= -\frac{\cos\phi}{\tan\phi}\left[\partial_\phi\left(\frac{\sec^2\phi}{\sec\phi}\right) + \partial_\theta\left(\frac{0}{\tan\phi}\right)\right]$$

$$= -\frac{\cos\phi}{\tan\phi}\left[\partial_\phi \sec\phi\right] = -\frac{\cos\phi}{\tan\phi}[\sec\phi\tan\phi]$$

$$= -\cos\phi\sec\phi = -1$$

This is what we wanted.