Needham, VDGF, Chapter 7, Exercise 24, p. 90
G. Palmer, July 2, 2022, 4 pm PDT

24. Amplitwist of z^m. *Generalize the argument in [4.6] to deduce that the amplitwist of* z^m *is given by the same formula as in real calculus:* $(z^m)' = mz^{m-1}$; *but now we can see why it's true!*

This is a revision of the suggested exercise on p. 44, which appears in the notes for this chapter: **P. 44**. *It is straightforward [exercise] to generalize this geometric argument to deduce that* $(z^m)' = mz^{m-1}$.

Let α = amplification, τ = twist, and "$\alpha\tau$" = amplitwist.

Since a radial edge through z at angle θ maps to a radial edge through z^m at angle mθ, the edge undergoes a rotation (twist) of

$$\tau = \mathrm{m}\theta - \theta = (m-1)\,\theta.$$

Since angles are multiplied by m, the image arc will subtend angle m$\delta\theta$, and since it now lies on a circle of radius r^n, the length of this image edge is ultimately $r^n(\mathrm{m}\delta\theta)$. The quotient is

$$r^m(\mathrm{m}\delta\theta)/(\mathrm{r}\delta\theta) = \mathrm{mr}^{m-1}$$

$$\left(\text{image edge} \asymp \mathrm{mr}^{m-1}(\text{original edge}) \Longrightarrow \alpha = \mathrm{mr}^{m-1}\right)$$

$$(z^m)' = \text{"}\alpha\tau\text{" of } z^m = \alpha \mathrm{e}^{\mathrm{i}\tau} = \mathrm{mr}^{m-1}\, e^{i(m-1)\,\theta} = \mathrm{mz}^{m-1}$$