Needham, VDGF, Chapter 7, Exercise 21, p. 89.
G. Palmer, June 22, 2022, 12:10 pm PDT

21. Conformal Curvature Formula. *Show that in the case of conformal mapping with metric (4.14) the general curvature formula (4.10) reduces to (4.16).*

$$d\hat{s}^2 = \Lambda^2(\mathrm{du}^2 + \mathrm{dv}^2) \qquad (4.14)$$

$$\mathcal{K} = -\frac{1}{\mathrm{AB}}\left(\partial_v\left[\frac{\partial_v A}{B}\right] + \partial_u\left[\frac{\partial_u B}{A}\right]\right) \qquad (4.10)$$

$$\mathcal{K} = -\frac{\nabla^2 \ln\Lambda}{\Lambda^2} \qquad (4.16)$$

Let A = B = Λ and substitute Λ for A and B into (4.10).

$$\mathcal{K} = -\frac{1}{\Lambda^2}\left(\partial_v\left[\frac{\partial_v \Lambda}{\Lambda}\right] + \partial_u\left[\frac{\partial_u \Lambda}{\Lambda}\right]\right)$$

$$= -\frac{1}{\Lambda^2}\left(\partial_v[\partial_v \ln\Lambda] + \partial_u[\partial_u \ln\Lambda]\right) = -\frac{1}{\Lambda^2}\left(\partial_v^2 \ln\Lambda + \partial_u^2 \ln\Lambda\right)$$

$$= -\frac{\nabla^2 \ln\Lambda}{\Lambda^2}$$

This was a suggested [exercise] on p. 41.