Needham, VDGF, Chapter 7, Exercise 18, p. 88.

G. Palmer, June 20, 2022, 12 noon PDT

*18.* ***The pseudosphere has finite area!*** *Use (5.5) and (4.12) to show that the infinite pseudosphere of radius R has finite area $2\pi R^2$....*

$$d\hat{s} = \frac{R\,\mathrm{ds}}{y} = \frac{R\sqrt{\mathrm{dx}^2+\mathrm{dy}^2}}{y} \qquad (5.5)$$

$$\mathrm{d}\mathcal{A} = \mathrm{AB\,du\,dv} \qquad (4.12)$$

Using (5.5) didn't work out for me, but I was able to reach the correct result using (5.2) and (4.12).*

$$d\hat{s}^2 = X^2\,\mathrm{dx}^2 + \mathrm{d}\sigma^2 = \left(\mathrm{Re}^{-\sigma/R}\right)^2 \mathrm{dx}^2 + \mathrm{d}\sigma^2 \qquad (5.2)$$

Using (4.12), let $A = X = \mathrm{Re}^{-\sigma/R}$ and let B = 1, where dx = du and d$\sigma$ = dv.

$$\mathrm{d}\mathcal{A} = \mathrm{Re}^{-\sigma/R}\,\mathrm{dx}\,\mathrm{d}\sigma$$

$$\mathcal{A} = R\int_0^\infty\int_0^{2\pi} e^{-\sigma/R}\,dx\,d\sigma = \left[-R^2 e^{-\sigma/R}\right]_0^\infty [x]_0^{2\pi}$$

$$= \left(-\mathrm{R}^2\,\mathrm{e}^{-\infty/\mathrm{R}} + \mathrm{R}^2\,\mathrm{e}^{-0/\mathrm{R}}\right)\ (2\pi - 0)$$

$$= \left(0 + \mathrm{R}^2\right) \times 2\pi - 0$$

$$= 2\pi\mathrm{R}^2$$

*My first attempt using (5.5) failed because I forgot that $y_{\mathrm{min}} = 1$ on the map of the pseudosphere. Note that the exercise requests a calculation based on variables in the hyperbolic plane, but my answer is based on variables on the pseudosphere.