Needham, VDGF, Chapter 39, Exercise 5, p. 466.
Palmer, June 25, 2026, 7:40 am PT

5. Tensor Contractions*. (From Schutz (1980).) How many different* $\left\{ \begin{matrix} 2 \\ 1 \end{matrix} \right\}$ *tensors can be made by contraction on pairs of indices of the* $\left\{ \begin{matrix} 3 \\ 2 \end{matrix} \right\}$ *tensor* $Q^{ijk}{}_{lm}$*? How many* $\left\{ \begin{matrix} 1 \\ 0 \end{matrix} \right\}$ *tensors (i.e., vectors) can be made by a second contraction?*

If $\mathbf{T}(\boldsymbol{\alpha},\boldsymbol{\beta},\boldsymbol{\gamma} \,||\, \mathbf{v}, \mathbf{w})$ is a tensor with valence $\left\{ \begin{matrix} 3 \\ 2 \end{matrix} \right\}$, one can reduce it as follows:

1st Reduction to {2 1}		**2nd Reduction to {1 0}**
$\mathbf{T}(\boldsymbol{\alpha}, \boldsymbol{\beta} \,||\, \mathbf{v})$	$\longrightarrow$	$\mathbf{T}(\boldsymbol{\alpha})$
$\mathbf{T}(\boldsymbol{\alpha}, \boldsymbol{\gamma} \,||\, \mathbf{v})$	$\longrightarrow$	$\mathbf{T}(\boldsymbol{\gamma})$
$\mathbf{T}(\boldsymbol{\beta}, \boldsymbol{\gamma} \,||\, \mathbf{v})$	$\longrightarrow$	$\mathbf{T}(\boldsymbol{\beta})$
$\mathbf{T}(\boldsymbol{\alpha}, \boldsymbol{\beta} \,||\, \mathbf{w})$	$\longrightarrow$	$\mathbf{T}(\boldsymbol{\alpha})$
$\mathbf{T}(\boldsymbol{\alpha}, \boldsymbol{\gamma} \,||\, \mathbf{w})$	$\longrightarrow$	$\mathbf{T}(\boldsymbol{\gamma})$
$\mathbf{T}(\boldsymbol{\beta}, \boldsymbol{\gamma} \,||\, \mathbf{w})$	$\longrightarrow$	$\mathbf{T}(\boldsymbol{\beta})$

There are only three distinct tensors in the second reduction.

Summarizing, six reductions are possible for a {3 2} tensor, and three more are possible in the second reduction to a {1, 0} tensor. So the answers to the question are six and three, respectively.