Needham, VDGF, Chapter 39, Exercise 2, p. 465.
Palmer. September 25, 2025, 7:30 am PDT

Out[1231]=

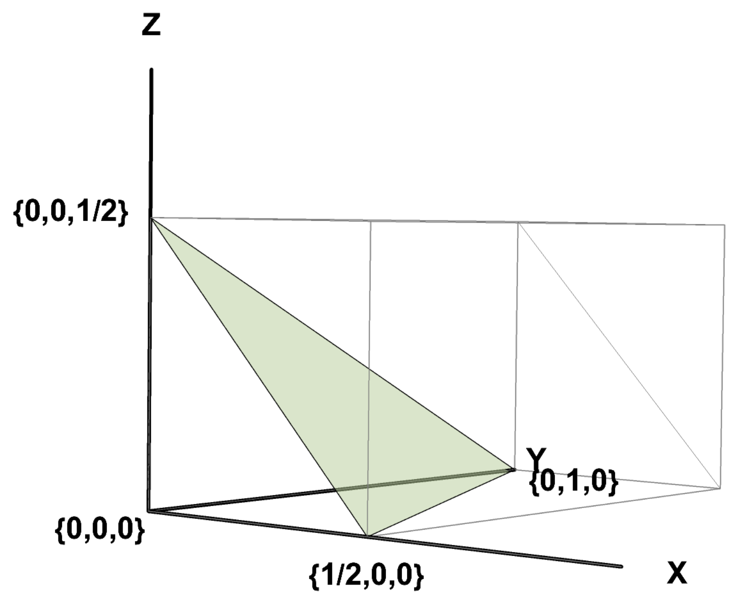

Figure 1. Small triangle generator for stack planes

Out[1232]=

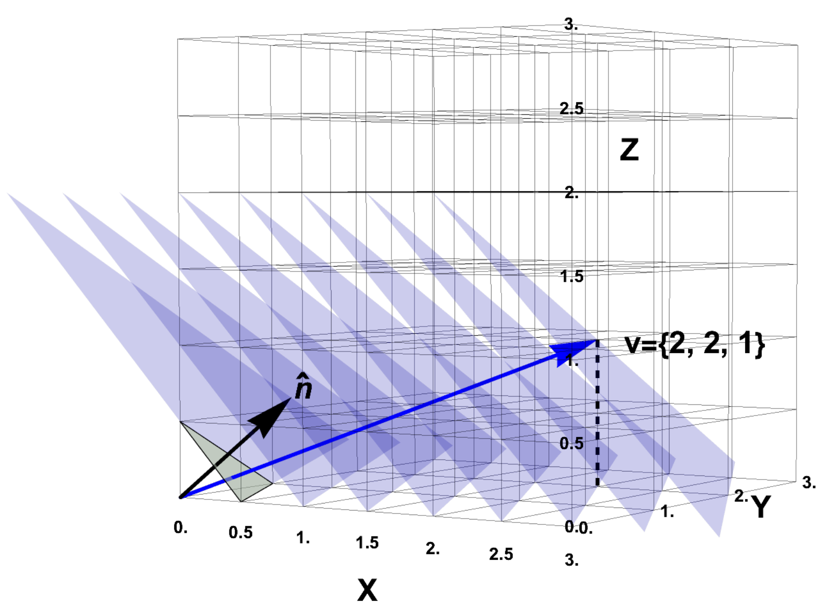

Figure 2. ω(v)=2dx+dy+2dz stack connected at points of intersection

2. In $\mathbb{R}^3$, give an explicit geometrical interpretation of the action $\boldsymbol{\omega}(\boldsymbol{v})$ of the 1-form $\boldsymbol{\omega} = 2\,\boldsymbol{d}x + \boldsymbol{d}y + 2\,\boldsymbol{d}z$ on the vector ***v****.*

The stack $\boldsymbol{\omega} = 2\,\boldsymbol{d}x + \boldsymbol{d}y + 2\,\boldsymbol{d}z$ has normal vector $(1/3)\,(2,\,1,\,2)^t$ with spacing of (1/3) between planar stacks and direction $(2,\,1,\,2)^t$.

Build $\boldsymbol{\omega}$ on the foundation of the stack shown in [32.9]. Adding orthogonal vertical stack lines on the vertical planes parallel to the xz-plane at intervals of (1/2) results in stacks of rectangular boxes (Figure 2). Since z in $\boldsymbol{\omega}$ is multiplied by 2, one can compress the stack vertically by adding (1/2) increments to z.

Connect the ends of the first stack of [32.9] to the point {0,0,1}, creating a triangle that defines the plane of the stack. This is the green triangle in Figure 1. The sides of the triangle form new diagonal lines connecting the two points {{1/2, 0, 0}, {0, 0, 1/2}} and connecting the two points {{0, 1, 0}, {0, 0, 1/2}}.

This procedure illustrates the geometric meaning of both multiplication and addition in $\boldsymbol{\omega} = 2\,\boldsymbol{d}x + \boldsymbol{d}y + 2\,\boldsymbol{d}z$.

The green triangle in Figure 1 is then multiplied by a constant to expand it into the planes of the blue stacks shown in Figure 2.

$\boldsymbol{\omega}(\boldsymbol{v})$ gives the number of planes pierced by the vector $(a,\ b,\ c)^t$ as $(\boldsymbol{\omega} = 2\,\boldsymbol{d}x + \boldsymbol{d}y + 2\,\boldsymbol{d}z)\,(a,\ b,\ c)^t = (2\,a + b + 2\,c)$. For example, the vector $(2,\,2,\,1)^t$, shown in blue in Figure 2 pierces $(2 \cdot 2 + 1 \cdot 2 + 2 \cdot 1) = 8$ planes in the blue stack.

The black arrow in Figure 2 is the unit normal. Real time rotation of the image as permitted within Mathematica confirms that the arrow is normal to the blue stacks.

In Figure 2, examining just the thin black framework, one can count 6 lines intersecting the x axis, but only 3 intersecting the y axis. The stacks along y appear to be compressed because of the viewing angle. But if seen from a high or low viewpoint, the sides of the rectangles are twice as long on the y axis as on the x axis. In other words, the density of stack lines spaced at equal intervals is twice as great on the x and z axes as on the y axis, as is described by the formula for $\boldsymbol{\omega}(\mathbf{v})$. All of the horizontal stacks shown fit within a 3x3x3 unit cube.