Needham, VDGAF, Chapter 3, Exercise 2, p. 24.
Palmer, November 17, 2021, 12 midnight PST

Out[1500]=

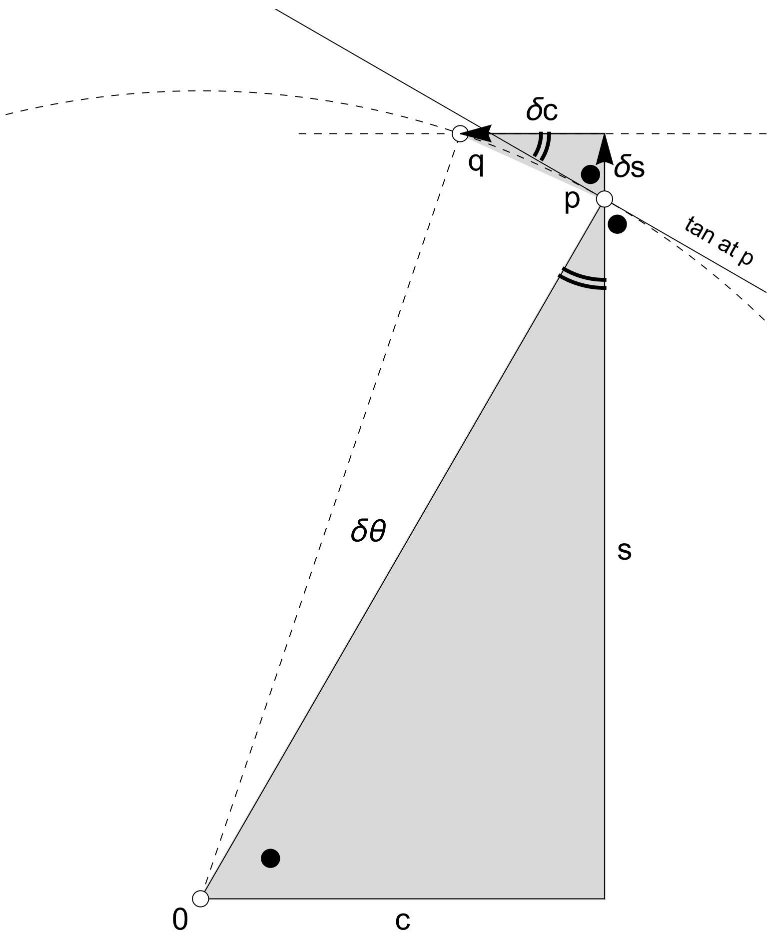


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Figure 1. Rotate p
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By emulating the Newtonian geometric argument in the prologue, instantly and simultaneously deduce that

$$\frac{ds}{d\theta} = c \qquad \text{and} \qquad \frac{dc}{d\theta} = -s.$$

As $\theta \longrightarrow 0$, the arc p to q approaches the tangent at p. As it does so, the small shaded triangle becomes similar to the large shaded triangle, as shown by the bold dots and arcs in the angles, where dots are complementary to arcs. That is, dot plus arc equals Pi/2. The arc of the unit circle between p and q has length $\delta\theta$, and is ultimately equal to the hypotenuse of the small triangle. Then $ds/d\theta \asymp \delta s/\delta\theta \asymp c/1 = c$. While $\delta\theta$ increases, δc decreases, so $dc/d\theta \asymp \delta c/\delta\theta \asymp -s$. Then,

As we get used to the idea of "ultimately equal" and of shapes which are "ultimately similar" we may very well feel confident enough to write the equality instantly, as Needham suggests, and write

$\frac{\mathrm{ds}}{\mathrm{d}\theta} = c$ [and $\frac{\mathrm{dc}}{\mathrm{d}\theta} = -s$]. (Vasco, p.c. VDGAF)