Needham, VDGAF, Chapter 3, Exercise 11, p. 26-27.
Palmer, December 14, 2021, 6:50 pm PST



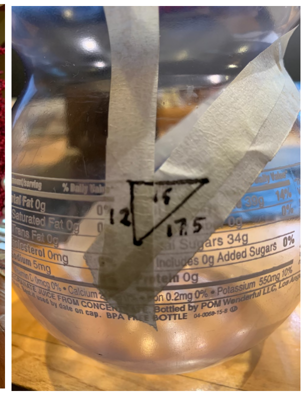

Photo 1 *Photo 2* *Photo 3*

11 Using the technique described in the footnote on page 14, or otherwise, manufacture narrow strips of sticky tape, ideally coloured masking tape. Then use (1.7) to conduct the following experiments on the surface of a vase of the approximate form shown in [11.7], on p. 127....

(i) Starting at a point on the horizontal circle of greatest radius, ρ_{max}, launch a geodesic straight up the vase, creating a meridian of the surface of revolution.

See photo 1.

(ii) Starting at the same point, launch several more geodesics in directions that make ever larger angles ψ with the meridians.

See photo 2. Only two more tapes were applied.

(iii) Note that beyond some critical angle ψ_c, the geodesics initially make their way up the vase, but then turn back and come down the vase!

See photo 2.

(iv) As best you can try to find the critical geodesic---the one that separates those that turn back from those that don't. ... measure its angle ψ_c.

The angle ψ_c was measured with a protractor to be 45 degrees (Pi/4). Since the protractor used is inflexible, the angle was measured by estimating the center line of the two tapes by viewing through the protractor held tangent to and against the starting point. The line of the critical angle appears straight when viewed through the transparent protractor. There is not much precision in this method of measurement.

Generally, I am measuring ψ_c as the clockwise angle from the first meridian anchored at the starting point on the widest horizontal of the bottom globe of the bottle or "vase". For the first leg of the geodesic traversal, the critical geodesic constitutes the border between the geodesics that rise and those that fall. It is only when it reaches the height of narrowest radius that it takes a horizontal trajectory and circles the waist of the vase.

(v) Let ρ_{max} be the maximum radius of the vase (at the launch point), and let ρ_{min} be the minimum radius, at the throat of the vase. By measuring diameters and dividing by 2, find these two radii as accurately as you can. Now verify that within experimental error,

$$\psi_c = \sin^{-1}[\rho_{min} / \rho_{max}]$$

By this formula, with $\rho_{min} = 4$ cm, $\rho_{max} = 5.89$ cm, $\psi_c \simeq .75$. Since the answer from (iv) was $\pi/4 = .785$, the two answers seem close enough within the experimental error. The difference is .035, which is about 4.6%.