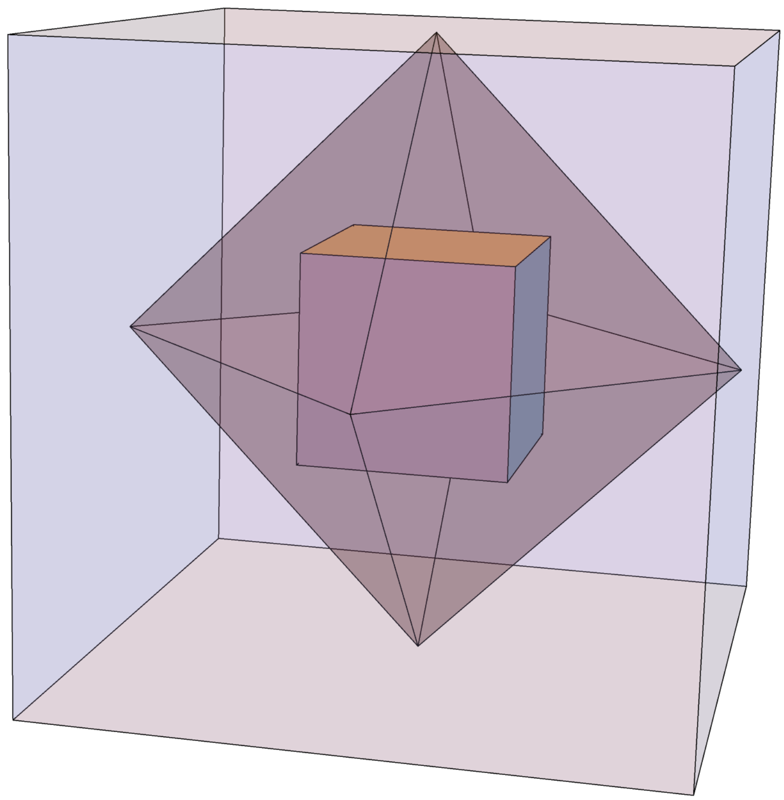
Needham, VDGAF, Chapter 20, Exercise 28, p. 227
Palmer, August 9 , 2023, 5 pm PST

Out[1592]=



Figue 1. Octahedron and two dual cubes

***28. Dual Polyhedra**. Place a vertex in the center of each face of a cube, and join them together to create an octohedron; this is the **dual** of the cube, each face having become a vertex, and each vertex having become a face. What if we keep going, by taking the dual of the octahedron, i.e., the dual of the cube?*

The dual of the octahedron is another cube, so there are infinite series of dual regular octahedrons and cubes. One might investigate the expansion of volume with each addition of a dual.

Out[1574]=

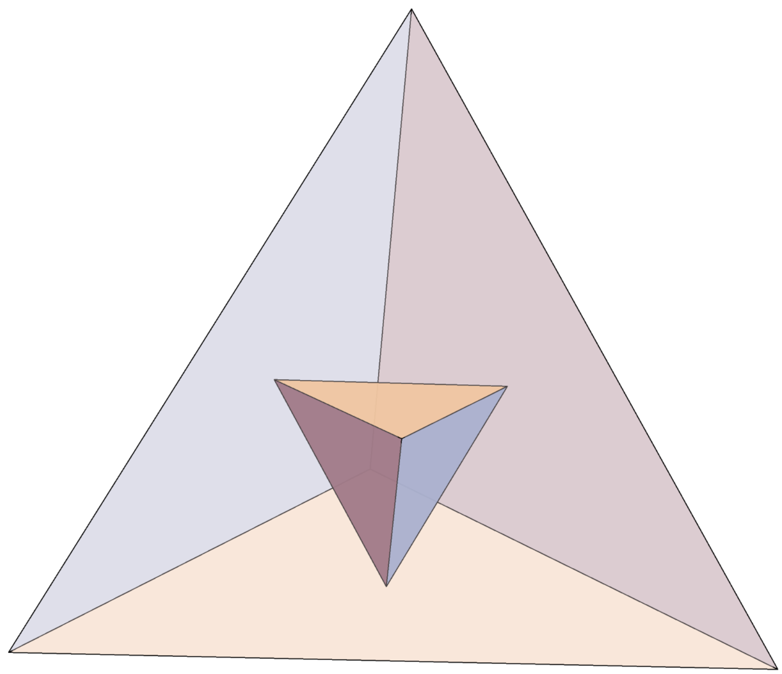


Figure 2. Dual tetrahedrons

With the assistance of [18.2], repeat this exercise for the three remaining Platonic solids.

The dual of a tetrahedron is another tetrahedron (Figure 2), so there is an infinite series of dual tetrahedrons.

Out[1901]=

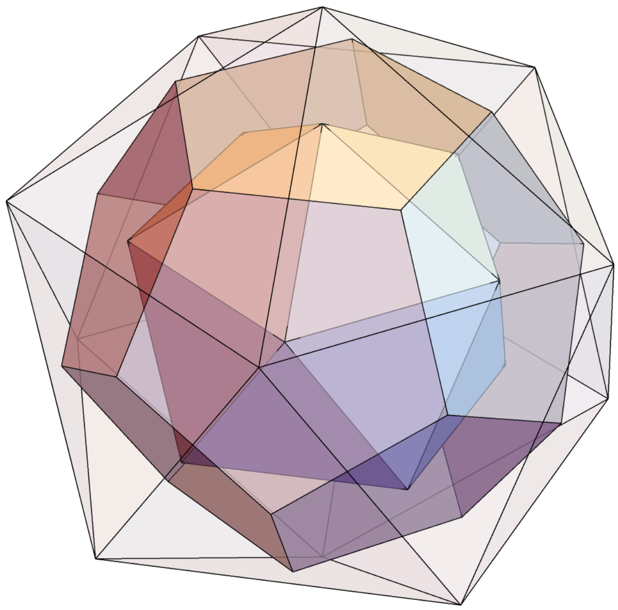


Figure 3. Dodecahedron between two icosahedron duals.

The dual fit depends on the outer polyhedron having the same number of faces as the inner has vertices. Then, if the vertices and faces are spaced regularly on their polyhedrons, one will fit inside the other. In this case, the icosahedron has 12 vertices and 20 faces, while the dodecahedron has 20 vertices and 12 faces, so with the proper scaling they form an infinite series of duals within duals (Figure 3). The tetrahedron has four vertices and four faces, so it can dual with itself (Figure 2).