Needham, VDGAF, Chapter 20, Exercise 24, p. 225
Palmer, July 24 , 2023, 12:30 pm PST

24. There Are Only Five Regular Polyhedra. *If each face of a regular polyhedron is a regular n-gon, and m such faces meet at each vertex, use the following steps to show that the only possibilities are the five Platonic solids illustrated in [18.2], namely , (n, m) = (3,3) or (4,3) or (3,4) or (5,3) or (3, 5).*

(i) Explain why $m \geq 3$.

$m \geq 3$ because two faces cannot define a vertex.

(ii) If θ_n is the internal angle of the regular n-gon face, explain why $m\theta_n < 2\pi$. (Hint: Imagine cutting out the m faces that meet at a vertex v, now cut along one of the edges containing v, and finally flatten out the surface onto a plane.)

If $m\theta_n = 2\pi$, the faces would form a flat surface, so they would not form a 3D vertex as needed for a polyhedron. If $m\theta_n > 2\pi$, they would overlap when flattened, so they could not form the vertex of a regular polyhedron.

(iii) Write down θ_n for n = 3, 4, 5, 6 and exhaust the possibilities.

n	θ_n	Polyhedron Faces
3	$\pi/3$	3, 8, 20
4	$\pi/2$	6
5	$3\pi/5$	12
6	$2\pi/3$	---

I don't understand the phrase "exhaust the possibilities" in this context. Each regular n-gon determines a single possibility of θ_n. Only three different n-gons are found in the five Platonic Solids, so the possibilities for θ_n are exhausted by $\pi/3$, $\pi/2$, and $3\pi/5$.