

Needham, VDGf, Chapter 20, Exercise 11, p. 221.

Palmer, February 26, 2022, 1:45 am PT

11. Why Paper Folds into a Straight Line. We take it for granted that when we fold a piece of paper, the fold automatically forms itself into a straight line, but why does this happen? Show (without any calculation) that this is a direct consequence of the Theorema Egregium!...

The Theorema Egregium states: The extrinsic curvature $\mathcal{K} = \left(\delta \tilde{\mathcal{A}} / \delta \mathcal{A} \right)$ is invariant under isometries of S and therefore belongs to the intrinsic geometry of S . More explicitly, while the principle curvatures individually depend on the shape of the surface in space, their product does not: $k_1 k_2$ is invariant under isometries.

Wikipedia uses more specific language. The isometries must be local.

In modern mathematical terminology, the theorem may be stated as follows:

The Gaussian curvature of a surface is invariant under **local** isometry. [emphasis added]

(https://en.wikipedia.org/wiki/Theorema_Egregium)

Answer: Paper is a rigid substance with local isometry. Stretching a paper surface is impossible by construction. In general, stretching in any direction would destroy the local isometry of the surface. Therefore *the fold automatically forms itself into a straight line*. This is the only mathematical fold that doesn't stretch.