Needham, VDGAF, Chapter 21, Exercise 9, p. 220-221.
Palmer, January 26, 2022, 4:45 pm PT

Out[274]=

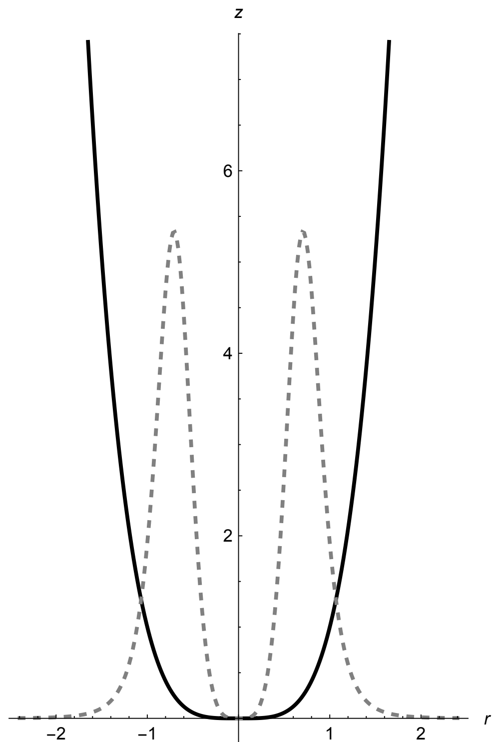

Figure 1. z = r^4
$\mathcal{K}$ plotted in dashed gray

10. In the text we noted that it is visually evident that the surface z = r^4 has a planar point (with $\mathcal{K}$ = 0) at the origin, and that the surface has a positive curvature everywhere else. Prove this is correct by using the formula in the previous question to calculate $\mathcal{K}$ (r).

Formula from previous question: $\mathcal{K}(r) = \frac{f'(r)\, f''(r)}{r\left\{1+[f'(r)]^2\right\}^2}$

$f'(r) = 4\, r^3$ $\quad\quad\quad\quad\quad\quad$ $f''(r) = 12\, r^2$

$$\mathcal{K}(r) = \frac{4\, r^3\, 12\, r^2}{r\left\{1+[4\, r^3]^2\right\}^2} = \frac{48\, r^5}{r\{1+4\, r^6\}^2} = \frac{48\, r^4}{\{1+4\, r^6\}^2}$$

$\mathcal{K}$(0) =0, and r ≥ 0 by definition. Since r is the only variable and no other minus signs appear in the equation, $\mathcal{K}$(r) is clearly positive for all r > 0, as shown in Figure 1. The proposition is proven to be correct. The numerator expands by r^5 while the denominator expands at the much higher rate of r^{12}, so for values of r > 1, as $r \longrightarrow \infty$, $\mathcal{K}$ plunges asymptotically in ever smaller fractions towards zero at a scale we cannot perceive, but never arrives. This appears to be an intensified and compacted version of Zeno's paradox. But $\mathcal{K}$ – rabbit must first travel away from the goal before it can approach it.