Needham, VDGF, Notes, Chapter 15.
Palmer, Mar 4, 2023

Out[3092]=

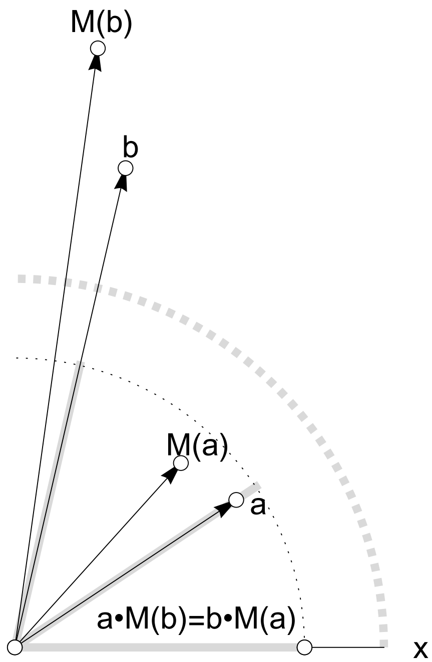


Figure 1. a•M(b) = b•M(a)
$\sigma_1$=0.75, $\sigma_2$=1.25
M={{$\sigma_1$,0},{0,$\sigma_2$}}

**P. 158**. (15.16) *Verify this both algebraically and geometrically: A linear transformation M is symmetric (i.e., $M^T = M$) iff*

$$\boldsymbol{a} \bullet M(\boldsymbol{b}) = \boldsymbol{b} \bullet M(\boldsymbol{a})$$

*for all **a** and **b**.*

It was given in (15.13) that a linear transformation M is symmetric if it has twist 0, which implies that it has orthogonal eigenvectors, and it was given in (15.15) that M is symmetric if it has orthogonal eigenvectors with eigenvalues $\sigma_1^2$ and $\sigma_2^2$. Then we can write:

$M = \begin{pmatrix} \sigma_1 & 0 \\ 0 & \sigma_2 \end{pmatrix}$, which is orthogonal because the dot product of the two columns vanishes.

$$M(\boldsymbol{a}) = \begin{pmatrix} \sigma_1 & 0 \\ 0 & \sigma_2 \end{pmatrix} . \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} = \begin{pmatrix} a_1\,\sigma_1 \\ a_2\,\sigma_2 \end{pmatrix} \Longrightarrow M(\boldsymbol{b}) = \begin{pmatrix} b_1\,\sigma_1 \\ b_2\,\sigma_2 \end{pmatrix}$$

$$\boldsymbol{a}\bullet M(\boldsymbol{b}) = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} \bullet \begin{pmatrix} b_1\,\sigma_1 \\ b_2\,\sigma_2 \end{pmatrix} = \begin{pmatrix} a_1\,b_1\,\sigma_1 \\ a_2\,b_2\,\sigma_2 \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} \bullet \begin{pmatrix} a_1\,\sigma_1 \\ a_2\,\sigma_2 \end{pmatrix} = \boldsymbol{b}\bullet M(\boldsymbol{a})$$

For a geometric validation, see Figure 1.